

Mathematica 11.3 Integration Test Results

Test results for the 1126 problems in "1.2.2.2 (d x)^m (a+b x^2+c x^4)^p.m"

Problem 8: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{a^2 + b + 2 a x^2 + x^4} dx$$

Optimal (type 3, 299 leaves, 9 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{-a+\sqrt{a^2+b}}-\sqrt{2}x}{\sqrt{a+\sqrt{a^2+b}}}\right]}{2\sqrt{2}\sqrt{a^2+b}\sqrt{a+\sqrt{a^2+b}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{-a+\sqrt{a^2+b}}+\sqrt{2}x}{\sqrt{a+\sqrt{a^2+b}}}\right]}{2\sqrt{2}\sqrt{a^2+b}\sqrt{a+\sqrt{a^2+b}}} - \frac{\text{Log}\left[\frac{\sqrt{a^2+b}-\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}x+x^2}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}}\right]}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}} + \frac{\text{Log}\left[\frac{\sqrt{a^2+b}+\sqrt{2}\sqrt{-a+\sqrt{a^2+b}}x+x^2}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}}\right]}{4\sqrt{2}\sqrt{a^2+b}\sqrt{-a+\sqrt{a^2+b}}}$$

Result (type 3, 81 leaves):

$$\frac{i \left(\frac{\text{ArcTan}\left[\frac{x}{\sqrt{a-i}\sqrt{b}}\right]}{\sqrt{a-i}\sqrt{b}} - \frac{\text{ArcTan}\left[\frac{x}{\sqrt{a+i}\sqrt{b}}\right]}{\sqrt{a+i}\sqrt{b}} \right)}{2\sqrt{b}}$$

Problem 10: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1 + a^2 + 2 a x^2 + x^4} dx$$

Optimal (type 3, 299 leaves, 9 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{-a+\sqrt{1+a^2}}-\sqrt{2}x}{\sqrt{a+\sqrt{1+a^2}}}\right]}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{-a+\sqrt{1+a^2}}+\sqrt{2}x}{\sqrt{a+\sqrt{1+a^2}}}\right]}{2\sqrt{2}\sqrt{1+a^2}\sqrt{a+\sqrt{1+a^2}}} - \frac{\text{Log}\left[\frac{\sqrt{1+a^2}-\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}x+x^2}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}}\right]}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}} + \frac{\text{Log}\left[\frac{\sqrt{1+a^2}+\sqrt{2}\sqrt{-a+\sqrt{1+a^2}}x+x^2}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}}\right]}{4\sqrt{2}\sqrt{1+a^2}\sqrt{-a+\sqrt{1+a^2}}}$$

Result (type 3, 52 leaves):

$$-\frac{1}{2} i \left(\frac{\text{ArcTan}\left[\frac{x}{\sqrt{-i+a}}\right]}{\sqrt{-i+a}} - \frac{\text{ArcTan}\left[\frac{x}{\sqrt{i+a}}\right]}{\sqrt{i+a}} \right)$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{4-5x^2+x^4} dx$$

Optimal (type 3, 17 leaves, 3 steps):

$$-\frac{1}{6} \text{ArcTanh}\left[\frac{x}{2}\right] + \frac{\text{ArcTanh}[x]}{3}$$

Result (type 3, 37 leaves):

$$-\frac{1}{6} \text{Log}[1-x] + \frac{1}{12} \text{Log}[2-x] + \frac{1}{6} \text{Log}[1+x] - \frac{1}{12} \text{Log}[2+x]$$

Problem 13: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{9+5x^2+x^4} dx$$

Optimal (type 3, 67 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{1-2x}{\sqrt{11}}\right]}{6\sqrt{11}} + \frac{\text{ArcTan}\left[\frac{1+2x}{\sqrt{11}}\right]}{6\sqrt{11}} - \frac{1}{12} \text{Log}[3-x+x^2] + \frac{1}{12} \text{Log}[3+x+x^2]$$

Result (type 3, 91 leaves):

$$-\frac{i \text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}(5-i\sqrt{11})}}\right]}{\sqrt{\frac{11}{2}(5-i\sqrt{11})}} + \frac{i \text{ArcTan}\left[\frac{x}{\sqrt{\frac{1}{2}(5+i\sqrt{11})}}\right]}{\sqrt{\frac{11}{2}(5+i\sqrt{11})}}$$

Problem 14: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{1-x^2+x^4} dx$$

Optimal (type 3, 74 leaves, 9 steps):

$$-\frac{1}{2} \text{ArcTan}[\sqrt{3}-2x] + \frac{1}{2} \text{ArcTan}[\sqrt{3}+2x] - \frac{\text{Log}[1-\sqrt{3}x+x^2]}{4\sqrt{3}} + \frac{\text{Log}[1+\sqrt{3}x+x^2]}{4\sqrt{3}}$$

Result (type 3, 77 leaves):

$$\frac{1}{\sqrt{6}} i \left(\sqrt{-1-i\sqrt{3}} \text{ArcTan}\left[\frac{1-i\sqrt{3}}{2}x\right] - \sqrt{-1+i\sqrt{3}} \text{ArcTan}\left[\frac{1+i\sqrt{3}}{2}x\right] \right)$$

Problem 15: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{2 + 2x^2 + x^4} dx$$

Optimal (type 3, 176 leaves, 9 steps):

$$-\frac{1}{4} \sqrt{-1 + \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2(-1 + \sqrt{2})} - 2x}{\sqrt{2(1 + \sqrt{2})}}\right] + \frac{1}{4} \sqrt{-1 + \sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2(-1 + \sqrt{2})} + 2x}{\sqrt{2(1 + \sqrt{2})}}\right] -$$

$$\frac{\operatorname{Log}\left[\sqrt{2} - \sqrt{2(-1 + \sqrt{2})} x + x^2\right]}{8\sqrt{-1 + \sqrt{2}}} + \frac{\operatorname{Log}\left[\sqrt{2} + \sqrt{2(-1 + \sqrt{2})} x + x^2\right]}{8\sqrt{-1 + \sqrt{2}}}$$

Result (type 3, 41 leaves):

$$\frac{1}{4} \left((1 - i)^{3/2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1 - i}}\right] + (1 + i)^{3/2} \operatorname{ArcTan}\left[\frac{x}{\sqrt{1 + i}}\right] \right)$$

Problem 16: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2 + 5x^2 - 3x^4}} dx$$

Optimal (type 4, 10 leaves, 2 steps):

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -6\right]$$

Result (type 4, 65 leaves):

$$\frac{i \sqrt{1 - \frac{x^2}{2}} \sqrt{1 + 3x^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{3} x\right], -\frac{1}{6}\right]}{\sqrt{3} \sqrt{2 + 5x^2 - 3x^4}}$$

Problem 17: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2 + 4x^2 - 3x^4}} dx$$

Optimal (type 4, 48 leaves, 2 steps):

$$\sqrt{\frac{1}{6} (2 + \sqrt{10})} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1}{2} (-2 + \sqrt{10})} x\right], \frac{1}{3} (-7 - 2\sqrt{10})\right]$$

Result (type 4, 49 leaves):

$$\frac{i \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{1 + \sqrt{\frac{5}{2}} x}\right], \frac{1}{3}(-7 + 2\sqrt{10})\right]}{\sqrt{2 + \sqrt{10}}}$$

Problem 18: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2 + 3x^2 - 3x^4}} dx$$

Optimal (type 4, 48 leaves, 2 steps):

$$\sqrt{\frac{2}{-3 + \sqrt{33}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{6}{3 + \sqrt{33}}} x\right], \frac{1}{4}(-7 - \sqrt{33})\right]$$

Result (type 4, 53 leaves):

$$-i \sqrt{\frac{2}{3 + \sqrt{33}}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{6}{-3 + \sqrt{33}}} x\right], \frac{1}{4}(-7 + \sqrt{33})\right]$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2 + 2x^2 - 3x^4}} dx$$

Optimal (type 4, 44 leaves, 2 steps):

$$\frac{\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{3}{1 + \sqrt{7}}} x\right], \frac{1}{3}(-4 - \sqrt{7})\right]}{\sqrt{-1 + \sqrt{7}}}$$

Result (type 4, 49 leaves):

$$\frac{i \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{3}{-1 + \sqrt{7}}} x\right], \frac{1}{3}(-4 + \sqrt{7})\right]}{\sqrt{1 + \sqrt{7}}}$$

Problem 20: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2 + x^2 - 3x^4}} dx$$

Optimal (type 4, 12 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}[x], -\frac{3}{2}\right]}{\sqrt{2}}$$

Result (type 4, 63 leaves):

$$-\frac{i \sqrt{1-x^2} \sqrt{2+3x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{3}{2}} x\right], -\frac{2}{3}\right]}{\sqrt{3} \sqrt{2+x^2-3x^4}}$$

Problem 23: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-2x^2-3x^4}} dx$$

Optimal (type 4, 42 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{3}{-1+\sqrt{7}}} x\right], \frac{1}{3}(-4+\sqrt{7})\right]}{\sqrt{1+\sqrt{7}}}$$

Result (type 4, 51 leaves):

$$-\frac{i \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{3}{1+\sqrt{7}}} x\right], -\frac{4}{3}-\frac{\sqrt{7}}{3}\right]}{\sqrt{-1+\sqrt{7}}}$$

Problem 24: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-3x^2-3x^4}} dx$$

Optimal (type 4, 46 leaves, 2 steps):

$$\sqrt{\frac{2}{3+\sqrt{33}}} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{6}{-3+\sqrt{33}}} x\right], \frac{1}{4}(-7+\sqrt{33})\right]$$

Result (type 4, 55 leaves):

$$-i \sqrt{\frac{2}{-3+\sqrt{33}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{6}{3+\sqrt{33}}} x\right], -\frac{7}{4}-\frac{\sqrt{33}}{4}\right]$$

Problem 25: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-4x^2-3x^4}} dx$$

Optimal (type 4, 48 leaves, 2 steps):

$$\sqrt{\frac{1}{6}(-2+\sqrt{10})} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1}{2}(2+\sqrt{10})}x\right], \frac{1}{3}(-7+2\sqrt{10})\right]$$

Result (type 4, 49 leaves):

$$\frac{i \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-1+\sqrt{\frac{5}{2}}}x\right], \frac{1}{3}(-7-2\sqrt{10})\right]}{\sqrt{-2+\sqrt{10}}}$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2-5x^2-3x^4}} dx$$

Optimal (type 4, 18 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{3}x\right], -\frac{1}{6}\right]}{\sqrt{6}}$$

Result (type 4, 54 leaves):

$$\frac{\sqrt{1-3x^2}\sqrt{2+x^2} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{3}x\right], -\frac{1}{6}\right]}{\sqrt{6}\sqrt{2-5x^2-3x^4}}$$

Problem 27: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+7x^2-2x^4}} dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\sqrt{\frac{2}{-7+\sqrt{73}}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{2x}{\sqrt{7+\sqrt{73}}}\right], \frac{1}{12}(-61-7\sqrt{73})\right]$$

Result (type 4, 52 leaves):

$$-i \sqrt{\frac{2}{7+\sqrt{73}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{2x}{\sqrt{-7+\sqrt{73}}}\right], \frac{1}{12}(-61+7\sqrt{73})\right]$$

Problem 28: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+6x^2-2x^4}} dx$$

Optimal (type 4, 44 leaves, 2 steps):

$$\sqrt{\frac{1}{6} (3 + \sqrt{15})} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1}{3} (-3 + \sqrt{15})} x\right], -4 - \sqrt{15}\right]$$

Result (type 4, 43 leaves):

$$\frac{i \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{1 + \sqrt{\frac{5}{3}}} x\right], -4 + \sqrt{15}\right]}{\sqrt{3 + \sqrt{15}}}$$

Problem 29: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3 + 5x^2 - 2x^4}} dx$$

Optimal (type 4, 10 leaves, 2 steps):

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{3}}\right], -6\right]$$

Result (type 4, 65 leaves):

$$\frac{i \sqrt{1 - \frac{x^2}{3}} \sqrt{1 + 2x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} x\right], -\frac{1}{6}\right]}{\sqrt{2} \sqrt{3 + 5x^2 - 2x^4}}$$

Problem 30: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3 + 4x^2 - 2x^4}} dx$$

Optimal (type 4, 44 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{2 + \sqrt{10}}} x\right], \frac{1}{3} (-7 - 2\sqrt{10})\right]}{\sqrt{-2 + \sqrt{10}}}$$

Result (type 4, 51 leaves):

$$\frac{i \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{-2 + \sqrt{10}}} x\right], -\frac{7}{3} + \frac{2\sqrt{10}}{3}\right]}{\sqrt{2 + \sqrt{10}}}$$

Problem 31: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+3x^2-2x^4}} dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\frac{\sqrt{\frac{2}{-3+\sqrt{33}}}}{\sqrt{3+\sqrt{33}}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{2x}{\sqrt{3+\sqrt{33}}}\right], \frac{1}{4}(-7-\sqrt{33})\right]$$

Result (type 4, 50 leaves):

$$-i \frac{\sqrt{\frac{2}{3+\sqrt{33}}}}{\sqrt{-3+\sqrt{33}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{2x}{\sqrt{-3+\sqrt{33}}}\right], \frac{1}{4}(-7+\sqrt{33})\right]$$

Problem 32: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+2x^2-2x^4}} dx$$

Optimal (type 4, 44 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{1+\sqrt{7}}} x\right], \frac{1}{3}(-4-\sqrt{7})\right]}{\sqrt{-1+\sqrt{7}}}$$

Result (type 4, 49 leaves):

$$- \frac{i \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{-1+\sqrt{7}}} x\right], \frac{1}{3}(-4+\sqrt{7})\right]}{\sqrt{1+\sqrt{7}}}$$

Problem 35: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3-x^2-2x^4}} dx$$

Optimal (type 4, 12 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}[x], -\frac{2}{3}\right]}{\sqrt{3}}$$

Result (type 4, 65 leaves):

$$\frac{i \sqrt{1-x^2} \sqrt{3+2x^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{3}} x\right], -\frac{3}{2}\right]}{\sqrt{2} \sqrt{3-x^2-2x^4}}$$

Problem 36: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-2x^2-2x^4}} dx$$

Optimal (type 4, 42 leaves, 2 steps):

$$\frac{\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{2}{-1+\sqrt{7}}} x\right], \frac{1}{3}(-4+\sqrt{7})\right]}{\sqrt{1+\sqrt{7}}}$$

Result (type 4, 51 leaves):

$$\frac{i \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{1+\sqrt{7}}} x\right], -\frac{4}{3}-\frac{\sqrt{7}}{3}\right]}{\sqrt{-1+\sqrt{7}}}$$

Problem 37: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-3x^2-2x^4}} dx$$

Optimal (type 4, 43 leaves, 2 steps):

$$\sqrt{\frac{2}{3+\sqrt{33}}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{2x}{\sqrt{-3+\sqrt{33}}}\right], \frac{1}{4}(-7+\sqrt{33})\right]$$

Result (type 4, 52 leaves):

$$-i \sqrt{\frac{2}{-3+\sqrt{33}}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{2x}{\sqrt{3+\sqrt{33}}}\right], -\frac{7}{4}-\frac{\sqrt{33}}{4}\right]$$

Problem 38: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-4x^2-2x^4}} dx$$

Optimal (type 4, 44 leaves, 2 steps):

$$\frac{\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{2}{-2+\sqrt{10}}} x\right], \frac{1}{3}(-7+2\sqrt{10})\right]}{\sqrt{2+\sqrt{10}}}$$

Result (type 4, 51 leaves):

$$\frac{\text{EllipticF}\left[\text{ArcSinh}\left[\sqrt{\frac{2}{2+\sqrt{10}}}\,x\right], -\frac{7}{3}-\frac{2\sqrt{10}}{3}\right]}{\sqrt{-2+\sqrt{10}}}$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3-5x^2-2x^4}} dx$$

Optimal (type 4, 18 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{2}\,x\right], -\frac{1}{6}\right]}{\sqrt{6}}$$

Result (type 4, 54 leaves):

$$\frac{\sqrt{1-2x^2}\sqrt{3+x^2}\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{2}\,x\right], -\frac{1}{6}\right]}{\sqrt{6}\sqrt{3-5x^2-2x^4}}$$

Problem 40: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-6x^2-2x^4}} dx$$

Optimal (type 4, 42 leaves, 2 steps):

$$\sqrt{\frac{1}{6}(-3+\sqrt{15})}\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1}{3}(3+\sqrt{15})}\,x\right], -4+\sqrt{15}\right]$$

Result (type 4, 45 leaves):

$$\frac{\text{EllipticF}\left[\text{ArcSinh}\left[\sqrt{-1+\sqrt{\frac{5}{3}}}\,x\right], -4-\sqrt{15}\right]}{\sqrt{-3+\sqrt{15}}}$$

Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-7x^2-2x^4}} dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\sqrt{\frac{2}{7+\sqrt{73}}}\text{EllipticF}\left[\text{ArcSin}\left[\frac{2x}{\sqrt{-7+\sqrt{73}}}\right], \frac{1}{12}(-61+7\sqrt{73})\right]$$

Result (type 4, 52 leaves):

$$-i \sqrt{\frac{2}{-7+\sqrt{73}}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{2 x}{\sqrt{7+\sqrt{73}}}\right], \frac{1}{12}(-61-7 \sqrt{73})\right]$$

Problem 43: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2+4 x^2+3 x^4}} dx$$

Optimal (type 4, 141 leaves, 1 step):

$$\left(\sqrt{\frac{2-(2-\sqrt{10}) x^2}{2-(2+\sqrt{10}) x^2}} \sqrt{-2+(2+\sqrt{10}) x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{2^{3/4} \times 5^{1/4} x}{\sqrt{-2+(2+\sqrt{10}) x^2}}\right], \frac{1}{10}(5+\sqrt{10})\right] \right) / \left(2 \times 10^{1/4} \sqrt{\frac{1}{2-(2+\sqrt{10}) x^2}} \sqrt{-2+4 x^2+3 x^4} \right)$$

Result (type 4, 81 leaves):

$$- \left(\left(i \sqrt{2-4 x^2-3 x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-1+\sqrt{\frac{5}{2}}} x\right], \frac{1}{3}(-7-2 \sqrt{10})\right] \right) / \left(\sqrt{-2+\sqrt{10}} \sqrt{-2+4 x^2+3 x^4} \right) \right)$$

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2+3 x^2+3 x^4}} dx$$

Optimal (type 4, 146 leaves, 1 step):

$$\left(\frac{\sqrt{\frac{4 - (3 - \sqrt{33}) x^2}{4 - (3 + \sqrt{33}) x^2}} \sqrt{-4 + (3 + \sqrt{33}) x^2}}{\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2} 33^{1/4} x}{\sqrt{-4 + (3 + \sqrt{33}) x^2}}\right], \frac{1}{22} (11 + \sqrt{33})\right]} \right) /$$

$$\left(2 \sqrt{2} 33^{1/4} \sqrt{\frac{1}{4 - (3 + \sqrt{33}) x^2}} \sqrt{-2 + 3 x^2 + 3 x^4} \right)$$

Result (type 4, 83 leaves):

$$\frac{i \sqrt{4 - 6 x^2 - 6 x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{6}{3 + \sqrt{33}}} x\right], -\frac{7}{4} - \frac{\sqrt{33}}{4}\right]}{\sqrt{-3 + \sqrt{33}} \sqrt{-2 + 3 x^2 + 3 x^4}}$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2 + 2 x^2 + 3 x^4}} dx$$

Optimal (type 4, 141 leaves, 1 step):

$$\left(\frac{\sqrt{\frac{2 - (1 - \sqrt{7}) x^2}{2 - (1 + \sqrt{7}) x^2}} \sqrt{-2 + (1 + \sqrt{7}) x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2} 7^{1/4} x}{\sqrt{-2 + (1 + \sqrt{7}) x^2}}\right], \frac{1}{14} (7 + \sqrt{7})\right]} \right) /$$

$$\left(2 \times 7^{1/4} \sqrt{\frac{1}{2 - (1 + \sqrt{7}) x^2}} \sqrt{-2 + 2 x^2 + 3 x^4} \right)$$

Result (type 4, 83 leaves):

$$\frac{i \sqrt{2 - 2 x^2 - 3 x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{3}{1 + \sqrt{7}}} x\right], -\frac{4}{3} - \frac{\sqrt{7}}{3}\right]}{\sqrt{-1 + \sqrt{7}} \sqrt{-2 + 2 x^2 + 3 x^4}}$$

Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2 - x^2 + 3 x^4}} dx$$

Optimal (type 4, 65 leaves, 1 step):

$$\frac{\sqrt{-1+x^2} \sqrt{2+3x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{5}{2}} x}{\sqrt{-1+x^2}}\right], \frac{2}{5}\right]}{\sqrt{5} \sqrt{-2-x^2+3x^4}}$$

Result (type 4, 60 leaves):

$$\frac{i \sqrt{1-x^2} \sqrt{2+3x^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{3}{2}} x\right], -\frac{2}{3}\right]}{\sqrt{-6-3x^2+9x^4}}$$

Problem 49: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-2x^2+3x^4}} dx$$

Optimal (type 4, 148 leaves, 1 step):

$$\left(\sqrt{-2-(1-\sqrt{7})x^2} \sqrt{\frac{2+(1+\sqrt{7})x^2}{2+(1-\sqrt{7})x^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{2} 7^{1/4} x}{\sqrt{-2-(1-\sqrt{7})x^2}}\right], \frac{1}{14}(7-\sqrt{7})\right] \right) /$$

$$\left(2 \times 7^{1/4} \sqrt{\frac{1}{2+(1-\sqrt{7})x^2}} \sqrt{-2-2x^2+3x^4} \right)$$

Result (type 4, 81 leaves):

$$\frac{i \sqrt{2+2x^2-3x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{3}{-1+\sqrt{7}}} x\right], \frac{1}{3}(-4+\sqrt{7})\right]}{\sqrt{1+\sqrt{7}} \sqrt{-2-2x^2+3x^4}}$$

Problem 50: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-3x^2+3x^4}} dx$$

Optimal (type 4, 153 leaves, 1 step):

$$\left(\sqrt{-4 - (3 - \sqrt{33}) x^2} \sqrt{\frac{4 + (3 + \sqrt{33}) x^2}{4 + (3 - \sqrt{33}) x^2}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2} 33^{1/4} x}{\sqrt{-4 - (3 - \sqrt{33}) x^2}}\right], \frac{1}{22} (11 - \sqrt{33})\right] \right) / \\ \left(2 \sqrt{2} 33^{1/4} \sqrt{\frac{1}{4 + (3 - \sqrt{33}) x^2}} \sqrt{-2 - 3 x^2 + 3 x^4} \right)$$

Result (type 4, 81 leaves):

$$- \left(\left(i \sqrt{4 + 6 x^2 - 6 x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{6}{-3 + \sqrt{33}}} x\right], \frac{1}{4} (-7 + \sqrt{33})\right] \right) / \right. \\ \left. \left(\sqrt{3 + \sqrt{33}} \sqrt{-2 - 3 x^2 + 3 x^4} \right) \right)$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2 - 4 x^2 + 3 x^4}} dx$$

Optimal (type 4, 148 leaves, 1 step):

$$\left(\sqrt{-2 - (2 - \sqrt{10}) x^2} \sqrt{\frac{2 + (2 + \sqrt{10}) x^2}{2 + (2 - \sqrt{10}) x^2}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{3/4} \times 5^{1/4} x}{\sqrt{-2 - (2 - \sqrt{10}) x^2}}\right], \frac{1}{10} (5 - \sqrt{10})\right] \right) / \\ \left(2 \times 10^{1/4} \sqrt{\frac{1}{2 + (2 - \sqrt{10}) x^2}} \sqrt{-2 - 4 x^2 + 3 x^4} \right)$$

Result (type 4, 81 leaves):

$$- \left(\left(i \sqrt{2+4x^2-3x^4} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{1+\sqrt{\frac{5}{2}}} x \right], \frac{1}{3} (-7+2\sqrt{10}) \right] \right) / \right. \\ \left. \left(\sqrt{2+\sqrt{10}} \sqrt{-2-4x^2+3x^4} \right) \right)$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-5x^2+3x^4}} dx$$

Optimal (type 4, 63 leaves, 1 step):

$$\frac{\sqrt{-2+x^2} \sqrt{1+3x^2} \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{7} x}{\sqrt{-2+x^2}} \right], \frac{1}{7} \right]}{\sqrt{7} \sqrt{-2-5x^2+3x^4}}$$

Result (type 4, 65 leaves):

$$- \frac{i \sqrt{1-\frac{x^2}{2}} \sqrt{1+3x^2} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{3} x \right], -\frac{1}{6} \right]}{\sqrt{3} \sqrt{-2-5x^2+3x^4}}$$

Problem 53: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3+7x^2+2x^4}} dx$$

Optimal (type 4, 148 leaves, 1 step):

$$\left(\sqrt{\frac{6-(7-\sqrt{73})x^2}{6-(7+\sqrt{73})x^2}} \sqrt{-6+(7+\sqrt{73})x^2} \right. \\ \left. \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{2} 73^{1/4} x}{\sqrt{-6+(7+\sqrt{73})x^2}} \right], \frac{1}{146} (73+7\sqrt{73}) \right] \right) / \\ \left(2\sqrt{3} 73^{1/4} \sqrt{\frac{1}{6-(7+\sqrt{73})x^2}} \sqrt{-3+7x^2+2x^4} \right)$$

Result (type 4, 80 leaves):

$$- \left(\left(i \sqrt{6 - 14 x^2 - 4 x^4} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\frac{2 x}{\sqrt{7 + \sqrt{73}}} \right], \frac{1}{12} (-61 - 7 \sqrt{73}) \right] \right) / \left(\sqrt{-7 + \sqrt{73}} \sqrt{-3 + 7 x^2 + 2 x^4} \right) \right)$$

Problem 54: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3 + 6 x^2 + 2 x^4}} dx$$

Optimal (type 4, 148 leaves, 1 step):

$$\left(\frac{\sqrt{\frac{3 - (3 - \sqrt{15}) x^2}{3 - (3 + \sqrt{15}) x^2}} \sqrt{-3 + (3 + \sqrt{15}) x^2}}{\operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{2} 15^{1/4} x}{\sqrt{-3 + (3 + \sqrt{15}) x^2}} \right], \frac{1}{10} (5 + \sqrt{15}) \right]} \right) / \left(\sqrt{2} 3^{3/4} \times 5^{1/4} \sqrt{\frac{1}{3 - (3 + \sqrt{15}) x^2}} \sqrt{-3 + 6 x^2 + 2 x^4} \right)$$

Result (type 4, 77 leaves):

$$\frac{i \sqrt{3 - 6 x^2 - 2 x^4} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-1 + \sqrt{\frac{5}{3}}} x \right], -4 - \sqrt{15} \right]}{\sqrt{-3 + \sqrt{15}} \sqrt{-3 + 6 x^2 + 2 x^4}}$$

Problem 56: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3 + 4 x^2 + 2 x^4}} dx$$

Optimal (type 4, 148 leaves, 1 step):

$$\left(\sqrt{\frac{3 - (2 - \sqrt{10}) x^2}{3 - (2 + \sqrt{10}) x^2}} \sqrt{-3 + (2 + \sqrt{10}) x^2} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{2^{3/4} \times 5^{1/4} x}{\sqrt{-3 + (2 + \sqrt{10}) x^2}}\right], \frac{1}{10} (5 + \sqrt{10})\right] \right) / \\ \left(2^{3/4} \sqrt{3} 5^{1/4} \sqrt{\frac{1}{3 - (2 + \sqrt{10}) x^2}} \sqrt{-3 + 4 x^2 + 2 x^4} \right)$$

Result (type 4, 83 leaves):

$$\frac{i \sqrt{3 - 4 x^2 - 2 x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{2 + \sqrt{10}}} x\right], -\frac{7}{3} - \frac{2\sqrt{10}}{3}\right]}{\sqrt{-2 + \sqrt{10}} \sqrt{-3 + 4 x^2 + 2 x^4}}$$

Problem 57: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3 + 3 x^2 + 2 x^4}} dx$$

Optimal (type 4, 146 leaves, 1 step):

$$\left(\sqrt{\frac{6 - (3 - \sqrt{33}) x^2}{6 - (3 + \sqrt{33}) x^2}} \sqrt{-6 + (3 + \sqrt{33}) x^2} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2} 33^{1/4} x}{\sqrt{-6 + (3 + \sqrt{33}) x^2}}\right], \frac{1}{22} (11 + \sqrt{33})\right] \right) / \\ \left(2 \times 3^{3/4} \times 11^{1/4} \sqrt{\frac{1}{6 - (3 + \sqrt{33}) x^2}} \sqrt{-3 + 3 x^2 + 2 x^4} \right)$$

Result (type 4, 80 leaves):

$$\frac{i \sqrt{6 - 6 x^2 - 4 x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{2 x}{\sqrt{3 + \sqrt{33}}}\right], -\frac{7}{4} - \frac{\sqrt{33}}{4}\right]}{\sqrt{-3 + \sqrt{33}} \sqrt{-3 + 3 x^2 + 2 x^4}}$$

Problem 58: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3+2x^2+2x^4}} dx$$

Optimal (type 4, 143 leaves, 1 step):

$$\left(\frac{\sqrt{\frac{3-(1-\sqrt{7})x^2}{3-(1+\sqrt{7})x^2}} \sqrt{-3+(1+\sqrt{7})x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{2} 7^{1/4} x}{\sqrt{-3+(1+\sqrt{7})x^2}}\right], \frac{1}{14}(7+\sqrt{7})\right]}{\sqrt{6} 7^{1/4} \sqrt{\frac{1}{3-(1+\sqrt{7})x^2}} \sqrt{-3+2x^2+2x^4}} \right)$$

Result (type 4, 83 leaves):

$$\frac{i \sqrt{3-2x^2-2x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{1+\sqrt{7}}} x\right], -\frac{4}{3}-\frac{\sqrt{7}}{3}\right]}{\sqrt{-1+\sqrt{7}} \sqrt{-3+2x^2+2x^4}}$$

Problem 59: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3+x^2+2x^4}} dx$$

Optimal (type 4, 63 leaves, 1 step):

$$\frac{\sqrt{-1+x^2} \sqrt{3+2x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{5}{3}} x}{\sqrt{-1+x^2}}\right], \frac{3}{5}\right]}{\sqrt{5} \sqrt{-3+x^2+2x^4}}$$

Result (type 4, 63 leaves):

$$\frac{i \sqrt{1-x^2} \sqrt{3+2x^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{3}} x\right], -\frac{3}{2}\right]}{\sqrt{2} \sqrt{-3+x^2+2x^4}}$$

Problem 62: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3-2x^2+2x^4}} dx$$

Optimal (type 4, 150 leaves, 1 step):

$$\left(\sqrt{-3 - (1 - \sqrt{7}) x^2} \sqrt{\frac{3 + (1 + \sqrt{7}) x^2}{3 + (1 - \sqrt{7}) x^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{2} 7^{1/4} x}{\sqrt{-3 - (1 - \sqrt{7}) x^2}}\right], \frac{1}{14} (7 - \sqrt{7})\right] \right) /$$

$$\left(\sqrt{6} 7^{1/4} \sqrt{\frac{1}{3 + (1 - \sqrt{7}) x^2}} \sqrt{-3 - 2 x^2 + 2 x^4} \right)$$

Result (type 4, 81 leaves):

$$\frac{i \sqrt{3 + 2 x^2 - 2 x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{-1 + \sqrt{7}}} x\right], \frac{1}{3} (-4 + \sqrt{7})\right]}{\sqrt{1 + \sqrt{7}} \sqrt{-3 - 2 x^2 + 2 x^4}}$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3 - 3 x^2 + 2 x^4}} dx$$

Optimal (type 4, 153 leaves, 1 step):

$$\left(\sqrt{-6 - (3 - \sqrt{33}) x^2} \sqrt{\frac{6 + (3 + \sqrt{33}) x^2}{6 + (3 - \sqrt{33}) x^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{2} 33^{1/4} x}{\sqrt{-6 - (3 - \sqrt{33}) x^2}}\right], \frac{1}{22} (11 - \sqrt{33})\right] \right) /$$

$$\left(2 \times 3^{3/4} \times 11^{1/4} \sqrt{\frac{1}{6 + (3 - \sqrt{33}) x^2}} \sqrt{-3 - 3 x^2 + 2 x^4} \right)$$

Result (type 4, 78 leaves):

$$\frac{i \sqrt{6 + 6 x^2 - 4 x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{2 x}{\sqrt{-3 + \sqrt{33}}}\right], \frac{1}{4} (-7 + \sqrt{33})\right]}{\sqrt{3 + \sqrt{33}} \sqrt{-3 - 3 x^2 + 2 x^4}}$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3 - 4 x^2 + 2 x^4}} dx$$

Optimal (type 4, 155 leaves, 1 step):

$$\left(\sqrt{-3 - (2 - \sqrt{10}) x^2} \sqrt{\frac{3 + (2 + \sqrt{10}) x^2}{3 + (2 - \sqrt{10}) x^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{2^{3/4} \times 5^{1/4} x}{\sqrt{-3 - (2 - \sqrt{10}) x^2}}\right], \frac{1}{10} (5 - \sqrt{10})\right] \right) / \left(2^{3/4} \sqrt{3} 5^{1/4} \sqrt{\frac{1}{3 + (2 - \sqrt{10}) x^2}} \sqrt{-3 - 4 x^2 + 2 x^4} \right)$$

Result (type 4, 83 leaves):

$$\frac{i \sqrt{3 + 4 x^2 - 2 x^4} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{-2 + \sqrt{10}}} x\right], -\frac{7}{3} + \frac{2\sqrt{10}}{3}\right]}{\sqrt{2 + \sqrt{10}} \sqrt{-3 - 4 x^2 + 2 x^4}}$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3 - 5 x^2 + 2 x^4}} dx$$

Optimal (type 4, 63 leaves, 1 step):

$$\frac{\sqrt{-3 + x^2} \sqrt{1 + 2 x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{7} x}{\sqrt{-3 + x^2}}\right], \frac{1}{7}\right]}{\sqrt{7} \sqrt{-3 - 5 x^2 + 2 x^4}}$$

Result (type 4, 65 leaves):

$$\frac{i \sqrt{1 - \frac{x^2}{3}} \sqrt{1 + 2 x^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{2} x\right], -\frac{1}{6}\right]}{\sqrt{2} \sqrt{-3 - 5 x^2 + 2 x^4}}$$

Problem 66: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2 + 5 x^2 + 3 x^4}} dx$$

Optimal (type 4, 52 leaves, 1 step):

$$\frac{(1+x^2) \sqrt{\frac{2+3x^2}{1+x^2}} \text{EllipticF}\left[\text{ArcTan}[x], -\frac{1}{2}\right]}{\sqrt{2} \sqrt{2+5x^2+3x^4}}$$

Result (type 4, 58 leaves):

$$\frac{i \sqrt{1+x^2} \sqrt{2+3x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{3}{2}} x\right], \frac{2}{3}\right]}{\sqrt{6+15x^2+9x^4}}$$

Problem 67: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+4x^2+3x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{(2+\sqrt{6}x^2) \sqrt{\frac{2+4x^2+3x^4}{(2+\sqrt{6}x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{2}-\frac{1}{\sqrt{6}}\right]}{2 \times 6^{1/4} \sqrt{2+4x^2+3x^4}}$$

Result (type 4, 144 leaves):

$$\frac{\left(\left(i \sqrt{1-\frac{3x^2}{-2-i\sqrt{2}}} \sqrt{1-\frac{3x^2}{-2+i\sqrt{2}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{3}{-2-i\sqrt{2}}} x\right], \frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right] \right) \right)}{\left(\sqrt{3} \sqrt{-\frac{1}{-2-i\sqrt{2}}} \sqrt{2+4x^2+3x^4} \right)}$$

Problem 68: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+3x^2+3x^4}} dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{(2+\sqrt{6}x^2) \sqrt{\frac{2+3x^2+3x^4}{(2+\sqrt{6}x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{8}(4-\sqrt{6})\right]}{2 \times 6^{1/4} \sqrt{2+3x^2+3x^4}}$$

Result (type 4, 144 leaves):

$$\frac{\left(\left(i \sqrt{1-\frac{6x^2}{-3-i\sqrt{15}}} \sqrt{1-\frac{6x^2}{-3+i\sqrt{15}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{6}{-3-i\sqrt{15}}} x\right], \frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right] \right) \right)}{\left(\sqrt{6} \sqrt{-\frac{1}{-3-i\sqrt{15}}} \sqrt{2+3x^2+3x^4} \right)}$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+2x^2+3x^4}} dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\left((2 + \sqrt{6} x^2) \sqrt{\frac{2 + 2x^2 + 3x^4}{(2 + \sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{12} (6 - \sqrt{6})\right]\right) / \left(2 \times 6^{1/4} \sqrt{2 + 2x^2 + 3x^4}\right)$$

Result (type 4, 144 leaves):

$$- \left(\left(i \sqrt{1 - \frac{3x^2}{-1 - i\sqrt{5}}} \sqrt{1 - \frac{3x^2}{-1 + i\sqrt{5}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{3}{-1 - i\sqrt{5}}} x\right], \frac{-1 - i\sqrt{5}}{-1 + i\sqrt{5}}\right]\right) / \left(\sqrt{3} \sqrt{-\frac{1}{-1 - i\sqrt{5}}} \sqrt{2 + 2x^2 + 3x^4}\right) \right)$$

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+x^2+3x^4}} dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\left((2 + \sqrt{6} x^2) \sqrt{\frac{2 + x^2 + 3x^4}{(2 + \sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{24} (12 - \sqrt{6})\right]\right) / \left(2 \times 6^{1/4} \sqrt{2 + x^2 + 3x^4}\right)$$

Result (type 4, 142 leaves):

$$- \left(\left(i \sqrt{1 - \frac{6x^2}{-1 - i\sqrt{23}}} \sqrt{1 - \frac{6x^2}{-1 + i\sqrt{23}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{6}{-1 - i\sqrt{23}}} x\right], \frac{-1 - i\sqrt{23}}{-1 + i\sqrt{23}}\right]\right) / \left(\sqrt{6} \sqrt{-\frac{1}{-1 - i\sqrt{23}}} \sqrt{2 + x^2 + 3x^4}\right) \right)$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+3x^4}} dx$$

Optimal (type 4, 72 leaves, 1 step):

$$\frac{(2 + \sqrt{6} x^2) \sqrt{\frac{2+3x^4}{(2+\sqrt{6}x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{2}\right]}{2 \times 6^{1/4} \sqrt{2+3x^4}}$$

Result (type 4, 25 leaves):

$$-\left(-\frac{1}{6}\right)^{1/4} \text{EllipticF}\left[i \text{ArcSinh}\left[\left(-\frac{3}{2}\right)^{1/4} x\right], -1\right]$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-x^2+3x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\left((2 + \sqrt{6} x^2) \sqrt{\frac{2-x^2+3x^4}{(2+\sqrt{6}x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{24} (12 + \sqrt{6})\right] \right) / \left(2 \times 6^{1/4} \sqrt{2-x^2+3x^4} \right)$$

Result (type 4, 144 leaves):

$$-\left(\left(i \sqrt{1 - \frac{6x^2}{1-i\sqrt{23}}} \sqrt{1 - \frac{6x^2}{1+i\sqrt{23}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{6}{1-i\sqrt{23}}} x\right], \frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right] \right) / \left(\sqrt{6} \sqrt{-\frac{1}{1-i\sqrt{23}}} \sqrt{2-x^2+3x^4} \right) \right)$$

Problem 73: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-2x^2+3x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\left((2 + \sqrt{6} x^2) \sqrt{\frac{2-2x^2+3x^4}{(2+\sqrt{6}x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{12} (6 + \sqrt{6})\right] \right) / \left(2 \times 6^{1/4} \sqrt{2-2x^2+3x^4} \right)$$

Result (type 4, 144 leaves):

$$- \left(\left(i \sqrt{1 - \frac{3x^2}{1-i\sqrt{5}}} \sqrt{1 - \frac{3x^2}{1+i\sqrt{5}}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{3}{1-i\sqrt{5}}} x \right], \frac{1-i\sqrt{5}}{1+i\sqrt{5}} \right] \right) / \left(\sqrt{3} \sqrt{-\frac{1}{1-i\sqrt{5}}} \sqrt{2-2x^2+3x^4} \right) \right)$$

Problem 74: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-3x^2+3x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{(2 + \sqrt{6} x^2) \sqrt{\frac{2-3x^2+3x^4}{(2+\sqrt{6} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\left(\frac{3}{2} \right)^{1/4} x \right], \frac{1}{8} (4 + \sqrt{6}) \right]}{2 \times 6^{1/4} \sqrt{2-3x^2+3x^4}}$$

Result (type 4, 144 leaves):

$$- \left(\left(i \sqrt{1 - \frac{6x^2}{3-i\sqrt{15}}} \sqrt{1 - \frac{6x^2}{3+i\sqrt{15}}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{6}{3-i\sqrt{15}}} x \right], \frac{3-i\sqrt{15}}{3+i\sqrt{15}} \right] \right) / \left(\sqrt{6} \sqrt{-\frac{1}{3-i\sqrt{15}}} \sqrt{2-3x^2+3x^4} \right) \right)$$

Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2-4x^2+3x^4}} dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{(2 + \sqrt{6} x^2) \sqrt{\frac{2-4x^2+3x^4}{(2+\sqrt{6} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\left(\frac{3}{2} \right)^{1/4} x \right], \frac{1}{2} + \frac{1}{\sqrt{6}} \right]}{2 \times 6^{1/4} \sqrt{2-4x^2+3x^4}}$$

Result (type 4, 144 leaves):

$$- \left(\left(i \sqrt{1 - \frac{3x^2}{2-i\sqrt{2}}} \sqrt{1 - \frac{3x^2}{2+i\sqrt{2}}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{3}{2-i\sqrt{2}}} x \right], \frac{2-i\sqrt{2}}{2+i\sqrt{2}} \right] \right) / \left(\sqrt{3} \sqrt{-\frac{1}{2-i\sqrt{2}}} \sqrt{2-4x^2+3x^4} \right) \right)$$

Problem 78: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+9x^2+2x^4}} dx$$

Optimal (type 4, 110 leaves, 1 step):

$$\left(\sqrt{\frac{6+(9-\sqrt{57})x^2}{6+(9+\sqrt{57})x^2}} \left(6+(9+\sqrt{57})x^2\right) \right. \\ \left. \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{6}(9+\sqrt{57})}x\right], \frac{1}{4}(-19+3\sqrt{57})\right]\right) / \left(\sqrt{6(9+\sqrt{57})} \sqrt{3+9x^2+2x^4}\right)$$

Result (type 4, 97 leaves):

$$-\frac{1}{2\sqrt{3+9x^2+2x^4}} \\ + i \sqrt{\frac{-9+\sqrt{57}-4x^2}{-9+\sqrt{57}}} \sqrt{9+\sqrt{57}+4x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{2x}{\sqrt{9+\sqrt{57}}}\right], \frac{23}{4} + \frac{3\sqrt{57}}{4}\right]$$

Problem 79: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+8x^2+2x^4}} dx$$

Optimal (type 4, 110 leaves, 1 step):

$$\left(\sqrt{\frac{3+(4-\sqrt{10})x^2}{3+(4+\sqrt{10})x^2}} \left(3+(4+\sqrt{10})x^2\right) \right. \\ \left. \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{3}(4+\sqrt{10})}x\right], -\frac{2}{3}(5-2\sqrt{10})\right]\right) / \left(\sqrt{3(4+\sqrt{10})} \sqrt{3+8x^2+2x^4}\right)$$

Result (type 4, 98 leaves):

$$-\frac{1}{\sqrt{6+16x^2+4x^4}} \\ + i \sqrt{\frac{-4+\sqrt{10}-2x^2}{-4+\sqrt{10}}} \sqrt{4+\sqrt{10}+2x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{4+\sqrt{10}}}x\right], \frac{13}{3} + \frac{4\sqrt{10}}{3}\right]$$

Problem 80: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+7x^2+2x^4}} dx$$

Optimal (type 4, 60 leaves, 1 step):

$$\frac{\sqrt{\frac{3+x^2}{1+2x^2}} (1+2x^2) \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{2} x\right], \frac{5}{6}\right]}{\sqrt{6} \sqrt{3+7x^2+2x^4}}$$

Result (type 4, 61 leaves):

$$\frac{i \sqrt{3+x^2} \sqrt{1+2x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} x\right], \frac{1}{6}\right]}{\sqrt{6} \sqrt{3+7x^2+2x^4}}$$

Problem 81: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+6x^2+2x^4}} dx$$

Optimal (type 4, 104 leaves, 1 step):

$$\left(\frac{\sqrt{\frac{3+(3-\sqrt{3})x^2}{3+(3+\sqrt{3})x^2}} (3+(3+\sqrt{3})x^2) \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{\frac{1}{3}(3+\sqrt{3})} x\right], -1+\sqrt{3}\right]}{\sqrt{3(3+\sqrt{3})} \sqrt{3+6x^2+2x^4}} \right) /$$

Result (type 4, 90 leaves):

$$-\frac{1}{\sqrt{6+12x^2+4x^4}} i \sqrt{\frac{-3+\sqrt{3}-2x^2}{-3+\sqrt{3}}} \sqrt{3+\sqrt{3}+2x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{1-\frac{1}{\sqrt{3}}} x\right], 2+\sqrt{3}\right]$$

Problem 82: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+5x^2+2x^4}} dx$$

Optimal (type 4, 52 leaves, 1 step):

$$\frac{(1+x^2) \sqrt{\frac{3+2x^2}{1+x^2}} \text{EllipticF}\left[\text{ArcTan}[x], \frac{1}{3}\right]}{\sqrt{3} \sqrt{3+5x^2+2x^4}}$$

Result (type 4, 58 leaves):

$$\frac{i \sqrt{1+x^2} \sqrt{3+2x^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{2}{3}} x\right], \frac{3}{2}\right]}{\sqrt{6+10x^2+4x^4}}$$

Problem 83: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+4x^2+2x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{(3+\sqrt{6}x^2) \sqrt{\frac{3+4x^2+2x^4}{(3+\sqrt{6}x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{2}-\frac{1}{\sqrt{6}}\right]}{2 \times 6^{1/4} \sqrt{3+4x^2+2x^4}}$$

Result (type 4, 144 leaves):

$$\frac{-\left(\left(i \sqrt{1-\frac{2x^2}{-2-i\sqrt{2}}} \sqrt{1-\frac{2x^2}{-2+i\sqrt{2}}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{2}{-2-i\sqrt{2}}} x\right], \frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right]\right) \sqrt{2} \sqrt{-\frac{1}{-2-i\sqrt{2}}} \sqrt{3+4x^2+2x^4}\right)}{1}$$

Problem 84: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+3x^2+2x^4}} dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{(3+\sqrt{6}x^2) \sqrt{\frac{3+3x^2+2x^4}{(3+\sqrt{6}x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{8}(4-\sqrt{6})\right]}{2 \times 6^{1/4} \sqrt{3+3x^2+2x^4}}$$

Result (type 4, 142 leaves):

$$\frac{-\left(\left(i \sqrt{1-\frac{4x^2}{-3-i\sqrt{15}}} \sqrt{1-\frac{4x^2}{-3+i\sqrt{15}}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[2 \sqrt{-\frac{1}{-3-i\sqrt{15}}} x\right], \frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right]\right) \left(2 \sqrt{-\frac{1}{-3-i\sqrt{15}}} \sqrt{3+3x^2+2x^4}\right)\right)}{1}$$

Problem 85: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3+2x^2+2x^4}} dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\left((3 + \sqrt{6} x^2) \sqrt{\frac{3 + 2 x^2 + 2 x^4}{(3 + \sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{12} (6 - \sqrt{6})\right] \right) / \left(2 \times 6^{1/4} \sqrt{3 + 2 x^2 + 2 x^4} \right)$$

Result (type 4, 144 leaves):

$$- \left(\left(i \sqrt{1 - \frac{2 x^2}{-1 - i \sqrt{5}}} \sqrt{1 - \frac{2 x^2}{-1 + i \sqrt{5}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2}{-1 - i \sqrt{5}}} x\right], \frac{-1 - i \sqrt{5}}{-1 + i \sqrt{5}}\right] \right) / \left(\sqrt{2} \sqrt{-\frac{1}{-1 - i \sqrt{5}}} \sqrt{3 + 2 x^2 + 2 x^4} \right) \right)$$

Problem 86: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3 + x^2 + 2 x^4}} dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\left((3 + \sqrt{6} x^2) \sqrt{\frac{3 + x^2 + 2 x^4}{(3 + \sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{24} (12 - \sqrt{6})\right] \right) / \left(2 \times 6^{1/4} \sqrt{3 + x^2 + 2 x^4} \right)$$

Result (type 4, 140 leaves):

$$- \left(\left(i \sqrt{1 - \frac{4 x^2}{-1 - i \sqrt{23}}} \sqrt{1 - \frac{4 x^2}{-1 + i \sqrt{23}}} \text{EllipticF}\left[i \text{ArcSinh}\left[2 \sqrt{-\frac{1}{-1 - i \sqrt{23}}} x\right], \frac{-1 - i \sqrt{23}}{-1 + i \sqrt{23}}\right] \right) / \left(2 \sqrt{-\frac{1}{-1 - i \sqrt{23}}} \sqrt{3 + x^2 + 2 x^4} \right) \right)$$

Problem 87: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3 + 2 x^4}} dx$$

Optimal (type 4, 72 leaves, 1 step):

$$\frac{(3 + \sqrt{6} x^2) \sqrt{\frac{3 + 2 x^4}{(3 + \sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{2}\right]}{2 \times 6^{1/4} \sqrt{3 + 2 x^4}}$$

Result (type 4, 25 leaves):

$$-\left(-\frac{1}{6}\right)^{1/4} \text{EllipticF}\left[\text{i ArcSinh}\left[\left(-\frac{2}{3}\right)^{1/4} x\right], -1\right]$$

Problem 88: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-x^2+2x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\left((3+\sqrt{6}x^2) \sqrt{\frac{3-x^2+2x^4}{(3+\sqrt{6}x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{24}(12+\sqrt{6})\right] \right) / \\ \left(2 \times 6^{1/4} \sqrt{3-x^2+2x^4} \right)$$

Result (type 4, 142 leaves):

$$-\left(\left(\text{i} \sqrt{1-\frac{4x^2}{1-\text{i}\sqrt{23}}} \sqrt{1-\frac{4x^2}{1+\text{i}\sqrt{23}}} \text{EllipticF}\left[\text{i ArcSinh}\left[2 \sqrt{-\frac{1}{1-\text{i}\sqrt{23}}} x\right], \frac{1-\text{i}\sqrt{23}}{1+\text{i}\sqrt{23}}\right] \right) / \right. \\ \left. \left(2 \sqrt{-\frac{1}{1-\text{i}\sqrt{23}}} \sqrt{3-x^2+2x^4} \right) \right)$$

Problem 89: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-2x^2+2x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\left((3+\sqrt{6}x^2) \sqrt{\frac{3-2x^2+2x^4}{(3+\sqrt{6}x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{12}(6+\sqrt{6})\right] \right) / \\ \left(2 \times 6^{1/4} \sqrt{3-2x^2+2x^4} \right)$$

Result (type 4, 144 leaves):

$$-\left(\left(\text{i} \sqrt{1-\frac{2x^2}{1-\text{i}\sqrt{5}}} \sqrt{1-\frac{2x^2}{1+\text{i}\sqrt{5}}} \text{EllipticF}\left[\text{i ArcSinh}\left[\sqrt{-\frac{2}{1-\text{i}\sqrt{5}}} x\right], \frac{1-\text{i}\sqrt{5}}{1+\text{i}\sqrt{5}}\right] \right) / \right. \\ \left. \left(\sqrt{2} \sqrt{-\frac{1}{1-\text{i}\sqrt{5}}} \sqrt{3-2x^2+2x^4} \right) \right)$$

Problem 90: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-3x^2+2x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{(3 + \sqrt{6} x^2) \sqrt{\frac{3-3x^2+2x^4}{(3+\sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{8} (4 + \sqrt{6})\right]}{2 \times 6^{1/4} \sqrt{3-3x^2+2x^4}}$$

Result (type 4, 142 leaves):

$$-\left(\left(i \sqrt{1 - \frac{4x^2}{3-i\sqrt{15}}} \sqrt{1 - \frac{4x^2}{3+i\sqrt{15}}} \text{EllipticF}\left[i \text{ArcSinh}\left[2 \sqrt{-\frac{1}{3-i\sqrt{15}}} x\right], \frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right]\right) / \left(2 \sqrt{-\frac{1}{3-i\sqrt{15}}} \sqrt{3-3x^2+2x^4}\right)\right)$$

Problem 91: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-4x^2+2x^4}} dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{(3 + \sqrt{6} x^2) \sqrt{\frac{3-4x^2+2x^4}{(3+\sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{2} + \frac{1}{\sqrt{6}}\right]}{2 \times 6^{1/4} \sqrt{3-4x^2+2x^4}}$$

Result (type 4, 144 leaves):

$$-\left(\left(i \sqrt{1 - \frac{2x^2}{2-i\sqrt{2}}} \sqrt{1 - \frac{2x^2}{2+i\sqrt{2}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2}{2-i\sqrt{2}}} x\right], \frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right]\right) / \left(\sqrt{2} \sqrt{-\frac{1}{2-i\sqrt{2}}} \sqrt{3-4x^2+2x^4}\right)\right)$$

Problem 95: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3+7x^2-2x^4}} dx$$

Optimal (type 4, 19 leaves, 2 steps):

$$-\frac{\text{EllipticF}\left[\text{ArcCos}\left[\frac{x}{\sqrt{3}}\right], \frac{6}{5}\right]}{\sqrt{5}}$$

Result (type 4, 58 leaves):

$$\frac{\sqrt{1-2x^2} \sqrt{1-\frac{x^2}{3}} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{2}x\right], \frac{1}{6}\right]}{\sqrt{2} \sqrt{-3+7x^2-2x^4}}$$

Problem 97: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3+5x^2-2x^4}} dx$$

Optimal (type 4, 14 leaves, 2 steps):

$$-\text{EllipticF}\left[\text{ArcCos}\left[\sqrt{\frac{2}{3}}x\right], 3\right]$$

Result (type 4, 53 leaves):

$$\frac{\sqrt{3-2x^2} \sqrt{1-x^2} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{3}}x\right], \frac{3}{2}\right]}{\sqrt{-6+10x^2-4x^4}}$$

Problem 98: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3+4x^2-2x^4}} dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{\left(3+\sqrt{6}x^2\right) \sqrt{\frac{3-4x^2+2x^4}{\left(3+\sqrt{6}x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right], \frac{1}{2}+\frac{1}{\sqrt{6}}\right]}{2 \times 6^{1/4} \sqrt{-3+4x^2-2x^4}}$$

Result (type 4, 144 leaves):

$$-\left(\left(i \sqrt{1-\frac{2x^2}{2-i\sqrt{2}}} \sqrt{1-\frac{2x^2}{2+i\sqrt{2}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2}{2-i\sqrt{2}}}x\right], \frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right]\right) \sqrt{2} \sqrt{-\frac{1}{2-i\sqrt{2}}} \sqrt{-3+4x^2-2x^4}\right)$$

Problem 99: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3+3x^2-2x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{(3+\sqrt{6}x^2)\sqrt{\frac{3-3x^2+2x^4}{(3+\sqrt{6}x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right], \frac{1}{8}(4+\sqrt{6})\right]}{2 \times 6^{1/4} \sqrt{-3+3x^2-2x^4}}$$

Result (type 4, 142 leaves):

$$-\left(\left(i\sqrt{1-\frac{4x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{3+i\sqrt{15}}}\text{EllipticF}\left[i\text{ArcSinh}\left[2\sqrt{-\frac{1}{3-i\sqrt{15}}}x\right], \frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right]\right)/\right. \\ \left.\left(2\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{-3+3x^2-2x^4}\right)\right)$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3+2x^2-2x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\left(\frac{(3+\sqrt{6}x^2)\sqrt{\frac{3-2x^2+2x^4}{(3+\sqrt{6}x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4}x\right], \frac{1}{12}(6+\sqrt{6})\right]\right)/ \\ \left(2 \times 6^{1/4} \sqrt{-3+2x^2-2x^4}\right)$$

Result (type 4, 144 leaves):

$$-\left(\left(i\sqrt{1-\frac{2x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{1+i\sqrt{5}}}\text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{-\frac{2}{1-i\sqrt{5}}}x\right], \frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right]\right)/\right. \\ \left.\left(\sqrt{2}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{-3+2x^2-2x^4}\right)\right)$$

Problem 101: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3+x^2-2x^4}} dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\left((3 + \sqrt{6} x^2) \sqrt{\frac{3 - x^2 + 2 x^4}{(3 + \sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{24} (12 + \sqrt{6})\right] \right) / \left(2 \times 6^{1/4} \sqrt{-3 + x^2 - 2 x^4} \right)$$

Result (type 4, 140 leaves):

$$- \left(\left(i \sqrt{1 - \frac{4 x^2}{1 - i \sqrt{23}}} \sqrt{1 - \frac{4 x^2}{1 + i \sqrt{23}}} \text{EllipticF}\left[i \text{ArcSinh}\left[2 \sqrt{-\frac{1}{1 - i \sqrt{23}}} x\right], \frac{1 - i \sqrt{23}}{1 + i \sqrt{23}}\right] \right) / \left(2 \sqrt{-\frac{1}{1 - i \sqrt{23}}} \sqrt{-3 + x^2 - 2 x^4} \right) \right)$$

Problem 102: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3 - 2 x^4}} dx$$

Optimal (type 4, 72 leaves, 1 step):

$$\frac{(3 + \sqrt{6} x^2) \sqrt{\frac{3 + 2 x^4}{(3 + \sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{2}\right]}{2 \times 6^{1/4} \sqrt{-3 - 2 x^4}}$$

Result (type 4, 47 leaves):

$$\frac{\left(-\frac{1}{6}\right)^{1/4} \sqrt{3 + 2 x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[\left(-\frac{2}{3}\right)^{1/4} x\right], -1\right]}{\sqrt{-3 - 2 x^4}}$$

Problem 103: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3 - x^2 - 2 x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\left((3 + \sqrt{6} x^2) \sqrt{\frac{3 + x^2 + 2 x^4}{(3 + \sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{24} (12 - \sqrt{6})\right] \right) / \left(2 \times 6^{1/4} \sqrt{-3 - x^2 - 2 x^4} \right)$$

Result (type 4, 142 leaves):

$$- \left(\left(i \sqrt{1 - \frac{4x^2}{-1 - i\sqrt{23}}} \sqrt{1 - \frac{4x^2}{-1 + i\sqrt{23}}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[2 \sqrt{-\frac{1}{-1 - i\sqrt{23}}} x \right], \frac{-1 - i\sqrt{23}}{-1 + i\sqrt{23}} \right] \right) / \left(2 \sqrt{-\frac{1}{-1 - i\sqrt{23}}} \sqrt{-3 - x^2 - 2x^4} \right) \right)$$

Problem 104: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3 - 2x^2 - 2x^4}} dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\left((3 + \sqrt{6} x^2) \sqrt{\frac{3 + 2x^2 + 2x^4}{(3 + \sqrt{6} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\left(\frac{2}{3} \right)^{1/4} x \right], \frac{1}{12} (6 - \sqrt{6}) \right] \right) / (2 \times 6^{1/4} \sqrt{-3 - 2x^2 - 2x^4})$$

Result (type 4, 144 leaves):

$$- \left(\left(i \sqrt{1 - \frac{2x^2}{-1 - i\sqrt{5}}} \sqrt{1 - \frac{2x^2}{-1 + i\sqrt{5}}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{2}{-1 - i\sqrt{5}}} x \right], \frac{-1 - i\sqrt{5}}{-1 + i\sqrt{5}} \right] \right) / \left(\sqrt{2} \sqrt{-\frac{1}{-1 - i\sqrt{5}}} \sqrt{-3 - 2x^2 - 2x^4} \right) \right)$$

Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3 - 3x^2 - 2x^4}} dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{(3 + \sqrt{6} x^2) \sqrt{\frac{3 + 3x^2 + 2x^4}{(3 + \sqrt{6} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\left(\frac{2}{3} \right)^{1/4} x \right], \frac{1}{8} (4 - \sqrt{6}) \right]}{2 \times 6^{1/4} \sqrt{-3 - 3x^2 - 2x^4}}$$

Result (type 4, 142 leaves):

$$- \left(\left(i \sqrt{1 - \frac{4x^2}{-3 - i\sqrt{15}}} \sqrt{1 - \frac{4x^2}{-3 + i\sqrt{15}}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[2 \sqrt{-\frac{1}{-3 - i\sqrt{15}}} x \right], \frac{-3 - i\sqrt{15}}{-3 + i\sqrt{15}} \right] \right) / \left(2 \sqrt{-\frac{1}{-3 - i\sqrt{15}}} \sqrt{-3 - 3x^2 - 2x^4} \right) \right)$$

Problem 106: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3-4x^2-2x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{(3 + \sqrt{6} x^2) \sqrt{\frac{3+4x^2+2x^4}{(3+\sqrt{6}x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{2}{3}\right)^{1/4} x\right], \frac{1}{2} - \frac{1}{\sqrt{6}}\right]}{2 \times 6^{1/4} \sqrt{-3-4x^2-2x^4}}$$

Result (type 4, 144 leaves):

$$-\left(\left(i \sqrt{1 - \frac{2x^2}{-2-i\sqrt{2}}} \sqrt{1 - \frac{2x^2}{-2+i\sqrt{2}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{2}{-2-i\sqrt{2}}} x\right], \frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right]\right) / \left(\sqrt{2} \sqrt{-\frac{1}{-2-i\sqrt{2}}} \sqrt{-3-4x^2-2x^4}\right)\right)$$

Problem 107: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-3-5x^2-2x^4}} dx$$

Optimal (type 4, 53 leaves, 2 steps):

$$\frac{\sqrt{3+2x^2} \text{EllipticF}\left[\text{ArcTan}[x], \frac{1}{3}\right]}{\sqrt{3} \sqrt{-1-x^2} \sqrt{\frac{3+2x^2}{1+x^2}}}$$

Result (type 4, 63 leaves):

$$\frac{i \sqrt{1+x^2} \sqrt{3+2x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{3}} x\right], \frac{3}{2}\right]}{\sqrt{2} \sqrt{-3-5x^2-2x^4}}$$

Problem 108: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-2+6x^2-3x^4}} dx$$

Optimal (type 4, 42 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcCos}\left[\sqrt{\frac{3}{3+\sqrt{3}}} x\right], \frac{1}{2} (1 + \sqrt{3})\right]}{\sqrt{2} 3^{1/4}}$$

Result (type 4, 85 leaves):

$$\frac{\left(\sqrt{3 - \sqrt{3} - 3 x^2} \sqrt{2 + (-3 + \sqrt{3}) x^2} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{1}{2}(3 + \sqrt{3}) x}\right], 2 - \sqrt{3}\right] \right)}{\left(\sqrt{6} \sqrt{-2 + 6 x^2 - 3 x^4} \right)}$$

Problem 109: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-2 + 5 x^2 - 3 x^4}} dx$$

Optimal (type 4, 6 leaves, 2 steps):

$$-\text{EllipticF}[\text{ArcCos}[x], 3]$$

Result (type 4, 53 leaves):

$$\frac{\sqrt{2 - 3 x^2} \sqrt{1 - x^2} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{3}{2}} x\right], \frac{2}{3}\right]}{\sqrt{-6 + 15 x^2 - 9 x^4}}$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2 + 4 x^2 - 3 x^4}} dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{\left(2 + \sqrt{6} x^2 \right) \sqrt{\frac{2 - 4 x^2 + 3 x^4}{(2 + \sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{2} + \frac{1}{\sqrt{6}}\right]}{2 \times 6^{1/4} \sqrt{-2 + 4 x^2 - 3 x^4}}$$

Result (type 4, 144 leaves):

$$-\left(\left(i \sqrt{1 - \frac{3 x^2}{2 - i \sqrt{2}}} \sqrt{1 - \frac{3 x^2}{2 + i \sqrt{2}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{3}{2 - i \sqrt{2}}} x\right], \frac{2 - i \sqrt{2}}{2 + i \sqrt{2}}\right] \right) \right) / \left(\sqrt{3} \sqrt{-\frac{1}{2 - i \sqrt{2}}} \sqrt{-2 + 4 x^2 - 3 x^4} \right)$$

Problem 111: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2 + 3 x^2 - 3 x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{(2 + \sqrt{6} x^2) \sqrt{\frac{2-3x^2+3x^4}{(2+\sqrt{6}x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{8} (4 + \sqrt{6})\right]}{2 \times 6^{1/4} \sqrt{-2 + 3x^2 - 3x^4}}$$

Result (type 4, 144 leaves):

$$-\left(\left(i \sqrt{1 - \frac{6x^2}{3 - i\sqrt{15}}} \sqrt{1 - \frac{6x^2}{3 + i\sqrt{15}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{6}{3 - i\sqrt{15}}} x\right], \frac{3 - i\sqrt{15}}{3 + i\sqrt{15}}\right]\right) / \left(\sqrt{6} \sqrt{-\frac{1}{3 - i\sqrt{15}}} \sqrt{-2 + 3x^2 - 3x^4}\right)\right)$$

Problem 112: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2 + 2x^2 - 3x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{\left((2 + \sqrt{6} x^2) \sqrt{\frac{2 - 2x^2 + 3x^4}{(2 + \sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{12} (6 + \sqrt{6})\right]\right) / \left(2 \times 6^{1/4} \sqrt{-2 + 2x^2 - 3x^4}\right)}$$

Result (type 4, 144 leaves):

$$-\left(\left(i \sqrt{1 - \frac{3x^2}{1 - i\sqrt{5}}} \sqrt{1 - \frac{3x^2}{1 + i\sqrt{5}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{3}{1 - i\sqrt{5}}} x\right], \frac{1 - i\sqrt{5}}{1 + i\sqrt{5}}\right]\right) / \left(\sqrt{3} \sqrt{-\frac{1}{1 - i\sqrt{5}}} \sqrt{-2 + 2x^2 - 3x^4}\right)\right)$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2 + x^2 - 3x^4}} dx$$

Optimal (type 4, 88 leaves, 1 step):

$$\frac{\left((2 + \sqrt{6} x^2) \sqrt{\frac{2 - x^2 + 3x^4}{(2 + \sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{24} (12 + \sqrt{6})\right]\right) / \left(2 \times 6^{1/4} \sqrt{-2 + x^2 - 3x^4}\right)}$$

Result (type 4, 142 leaves):

$$- \left(\left(i \sqrt{1 - \frac{6x^2}{1 - i\sqrt{23}}} \sqrt{1 - \frac{6x^2}{1 + i\sqrt{23}}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{6}{1 - i\sqrt{23}}} x \right], \frac{1 - i\sqrt{23}}{1 + i\sqrt{23}} \right] \right) / \left(\sqrt{6} \sqrt{-\frac{1}{1 - i\sqrt{23}}} \sqrt{-2 + x^2 - 3x^4} \right) \right)$$

Problem 114: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2 - 3x^4}} dx$$

Optimal (type 4, 72 leaves, 1 step):

$$\frac{(2 + \sqrt{6} x^2) \sqrt{\frac{2+3x^4}{(2+\sqrt{6} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\left(\frac{3}{2} \right)^{1/4} x \right], \frac{1}{2} \right]}{2 \times 6^{1/4} \sqrt{-2 - 3x^4}}$$

Result (type 4, 47 leaves):

$$\frac{\left(-\frac{1}{6}\right)^{1/4} \sqrt{2 + 3x^4} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\left(-\frac{3}{2}\right)^{1/4} x \right], -1 \right]}{\sqrt{-2 - 3x^4}}$$

Problem 115: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2 - x^2 - 3x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\left((2 + \sqrt{6} x^2) \sqrt{\frac{2 + x^2 + 3x^4}{(2 + \sqrt{6} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\left(\frac{3}{2} \right)^{1/4} x \right], \frac{1}{24} (12 - \sqrt{6}) \right] \right) / \left(2 \times 6^{1/4} \sqrt{-2 - x^2 - 3x^4} \right)$$

Result (type 4, 144 leaves):

$$- \left(\left(i \sqrt{1 - \frac{6x^2}{-1 - i\sqrt{23}}} \sqrt{1 - \frac{6x^2}{-1 + i\sqrt{23}}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{6}{-1 - i\sqrt{23}}} x \right], \frac{-1 - i\sqrt{23}}{-1 + i\sqrt{23}} \right] \right) / \left(\sqrt{6} \sqrt{-\frac{1}{-1 - i\sqrt{23}}} \sqrt{-2 - x^2 - 3x^4} \right) \right)$$

Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-2x^2-3x^4}} dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{\left((2 + \sqrt{6} x^2) \sqrt{\frac{2 + 2x^2 + 3x^4}{(2 + \sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{12} (6 - \sqrt{6})\right]\right)}{2 \times 6^{1/4} \sqrt{-2-2x^2-3x^4}}$$

Result (type 4, 144 leaves):

$$-\left(\left(i \sqrt{1 - \frac{3x^2}{-1 - i\sqrt{5}}} \sqrt{1 - \frac{3x^2}{-1 + i\sqrt{5}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{3}{-1 - i\sqrt{5}}} x\right], \frac{-1 - i\sqrt{5}}{-1 + i\sqrt{5}}\right]\right) / \left(\sqrt{3} \sqrt{-\frac{1}{-1 - i\sqrt{5}}} \sqrt{-2-2x^2-3x^4}\right)\right)$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-3x^2-3x^4}} dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\frac{\left((2 + \sqrt{6} x^2) \sqrt{\frac{2+3x^2+3x^4}{(2+\sqrt{6} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{8} (4 - \sqrt{6})\right]\right)}{2 \times 6^{1/4} \sqrt{-2-3x^2-3x^4}}$$

Result (type 4, 144 leaves):

$$-\left(\left(i \sqrt{1 - \frac{6x^2}{-3 - i\sqrt{15}}} \sqrt{1 - \frac{6x^2}{-3 + i\sqrt{15}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{6}{-3 - i\sqrt{15}}} x\right], \frac{-3 - i\sqrt{15}}{-3 + i\sqrt{15}}\right]\right) / \left(\sqrt{6} \sqrt{-\frac{1}{-3 - i\sqrt{15}}} \sqrt{-2-3x^2-3x^4}\right)\right)$$

Problem 118: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-4x^2-3x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{(2 + \sqrt{6} x^2) \sqrt{\frac{2+4x^2+3x^4}{(2+\sqrt{6}x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{3}{2}\right)^{1/4} x\right], \frac{1}{2} - \frac{1}{\sqrt{6}}\right]}{2 \times 6^{1/4} \sqrt{-2 - 4x^2 - 3x^4}}$$

Result (type 4, 144 leaves):

$$-\left(\left(i \sqrt{1 - \frac{3x^2}{-2 - i\sqrt{2}}} \sqrt{1 - \frac{3x^2}{-2 + i\sqrt{2}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{3}{-2 - i\sqrt{2}}} x\right], \frac{-2 - i\sqrt{2}}{-2 + i\sqrt{2}}\right]\right) / \left(\sqrt{3} \sqrt{-\frac{1}{-2 - i\sqrt{2}}} \sqrt{-2 - 4x^2 - 3x^4}\right)\right)$$

Problem 119: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2 - 5x^2 - 3x^4}} dx$$

Optimal (type 4, 52 leaves, 2 steps):

$$\frac{\sqrt{-2 - 3x^2} \text{EllipticF}\left[\text{ArcTan}[x], -\frac{1}{2}\right]}{\sqrt{2} \sqrt{1 + x^2} \sqrt{\frac{2+3x^2}{1+x^2}}}$$

Result (type 4, 63 leaves):

$$\frac{i \sqrt{1 + x^2} \sqrt{2 + 3x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{3}{2}} x\right], \frac{2}{3}\right]}{\sqrt{3} \sqrt{-2 - 5x^2 - 3x^4}}$$

Problem 120: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2 + 5x^2 + 5x^4}} dx$$

Optimal (type 4, 92 leaves, 1 step):

$$\left(\left(2 + \sqrt{10} x^2\right) \sqrt{\frac{2 + 5x^2 + 5x^4}{(2 + \sqrt{10} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\left(\frac{5}{2}\right)^{1/4} x\right], \frac{1}{8} (4 - \sqrt{10})\right]\right) / \left(2 \times 10^{1/4} \sqrt{2 + 5x^2 + 5x^4}\right)$$

Result (type 4, 144 leaves):

$$- \left(\left(i \sqrt{1 - \frac{10 x^2}{-5 - i \sqrt{15}}} \sqrt{1 - \frac{10 x^2}{-5 + i \sqrt{15}}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{-\frac{10}{-5 - i \sqrt{15}}} x \right], \frac{-5 - i \sqrt{15}}{-5 + i \sqrt{15}} \right] \right) / \left(\sqrt{10} \sqrt{-\frac{1}{-5 - i \sqrt{15}}} \sqrt{2 + 5 x^2 + 5 x^4} \right) \right)$$

Problem 121: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2 + 5 x^2 + 4 x^4}} dx$$

Optimal (type 4, 90 leaves, 1 step):

$$\frac{(1 + \sqrt{2} x^2) \sqrt{\frac{2 + 5 x^2 + 4 x^4}{(1 + \sqrt{2} x^2)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[2^{1/4} x \right], \frac{1}{16} (8 - 5 \sqrt{2}) \right]}{2 \times 2^{3/4} \sqrt{2 + 5 x^2 + 4 x^4}}$$

Result (type 4, 147 leaves):

$$- \left(\left(i \sqrt{1 - \frac{8 x^2}{-5 - i \sqrt{7}}} \sqrt{1 - \frac{8 x^2}{-5 + i \sqrt{7}}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[2 \sqrt{-\frac{2}{-5 - i \sqrt{7}}} x \right], \frac{-5 - i \sqrt{7}}{-5 + i \sqrt{7}} \right] \right) / \left(2 \sqrt{2} \sqrt{-\frac{1}{-5 - i \sqrt{7}}} \sqrt{2 + 5 x^2 + 4 x^4} \right) \right)$$

Problem 122: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2 + 5 x^2 + 3 x^4}} dx$$

Optimal (type 4, 52 leaves, 1 step):

$$\frac{(1 + x^2) \sqrt{\frac{2 + 3 x^2}{1 + x^2}} \operatorname{EllipticF} \left[\operatorname{ArcTan} [x], -\frac{1}{2} \right]}{\sqrt{2} \sqrt{2 + 5 x^2 + 3 x^4}}$$

Result (type 4, 58 leaves):

$$- \frac{i \sqrt{1 + x^2} \sqrt{2 + 3 x^2} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{3}{2}} x \right], \frac{2}{3} \right]}{\sqrt{6 + 15 x^2 + 9 x^4}}$$

Problem 123: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2 + 5 x^2 + 2 x^4}} dx$$

Optimal (type 4, 58 leaves, 1 step):

$$\frac{\sqrt{\frac{2+x^2}{1+2x^2}} (1+2x^2) \text{EllipticF}\left[\text{ArcTan}\left[\sqrt{2} x\right], \frac{3}{4}\right]}{2 \sqrt{2+5x^2+2x^4}}$$

Result (type 4, 58 leaves):

$$\frac{i \sqrt{2+x^2} \sqrt{1+2x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} x\right], \frac{1}{4}\right]}{2 \sqrt{2+5x^2+2x^4}}$$

Problem 124: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5x^2+x^4}} dx$$

Optimal (type 4, 108 leaves, 1 step):

$$\left(\frac{\sqrt{\frac{4+(5-\sqrt{17})x^2}{4+(5+\sqrt{17})x^2}} (4+(5+\sqrt{17})x^2) \text{EllipticF}\left[\text{ArcTan}\left[\frac{1}{2} \sqrt{5+\sqrt{17}} x\right], \frac{1}{4} (-17+5\sqrt{17})\right]}{2 \sqrt{5+\sqrt{17}} \sqrt{2+5x^2+x^4}} \right) /$$

Result (type 4, 103 leaves):

$$-\left(\left(i \sqrt{5-\sqrt{17}+2x^2} \sqrt{5+\sqrt{17}+2x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{5+\sqrt{17}}} x\right], \frac{21}{4} + \frac{5\sqrt{17}}{4}\right] \right) / \left(\sqrt{2(5-\sqrt{17})} \sqrt{2+5x^2+x^4} \right) \right)$$

Problem 125: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5x^2-x^4}} dx$$

Optimal (type 4, 48 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{33}}} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{2}{5+\sqrt{33}}} x\right], \frac{1}{4} (-29-5\sqrt{33})\right]$$

Result (type 4, 55 leaves):

$$-i \sqrt{\frac{2}{5+\sqrt{33}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{2}{-5+\sqrt{33}}} x\right], -\frac{29}{4} + \frac{5\sqrt{33}}{4}\right]$$

Problem 126: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5x^2-2x^4}} dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{41}}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{2x}{\sqrt{5+\sqrt{41}}}\right], \frac{1}{8}(-33-5\sqrt{41})\right]$$

Result (type 4, 52 leaves):

$$-i \sqrt{\frac{2}{5+\sqrt{41}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{2x}{\sqrt{-5+\sqrt{41}}}\right], -\frac{33}{8} + \frac{5\sqrt{41}}{8}\right]$$

Problem 127: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2+5x^2-3x^4}} dx$$

Optimal (type 4, 10 leaves, 2 steps):

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{x}{\sqrt{2}}\right], -6\right]$$

Result (type 4, 65 leaves):

$$\frac{i \sqrt{1-\frac{x^2}{2}} \sqrt{1+3x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{3}x\right], -\frac{1}{6}\right]}{\sqrt{3} \sqrt{2+5x^2-3x^4}}$$

Problem 128: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5x^2-4x^4}} dx$$

Optimal (type 4, 49 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{57}}} \text{EllipticF}\left[\text{ArcSin}\left[2 \sqrt{\frac{2}{5+\sqrt{57}}} x\right], \frac{1}{16}(-41-5\sqrt{57})\right]$$

Result (type 4, 56 leaves):

$$-i \sqrt{\frac{2}{5+\sqrt{57}}} \text{EllipticF}\left[i \text{ArcSinh}\left[2 \sqrt{\frac{2}{-5+\sqrt{57}}} x\right], \frac{1}{16}(-41+5\sqrt{57})\right]$$

Problem 129: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5x^2-5x^4}} dx$$

Optimal (type 4, 48 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{65}}} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\frac{10}{5+\sqrt{65}}} x\right], \frac{1}{4}(-9-\sqrt{65})\right]$$

Result (type 4, 52 leaves):

$$-i \sqrt{\frac{2}{5+\sqrt{65}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{1}{2}\sqrt{5+\sqrt{65}} x\right], \frac{1}{4}(-9+\sqrt{65})\right]$$

Problem 130: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5x^2-6x^4}} dx$$

Optimal (type 4, 49 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{73}}} \text{EllipticF}\left[\text{ArcSin}\left[2\sqrt{\frac{3}{5+\sqrt{73}}} x\right], \frac{1}{24}(-49-5\sqrt{73})\right]$$

Result (type 4, 56 leaves):

$$-i \sqrt{\frac{2}{5+\sqrt{73}}} \text{EllipticF}\left[i \text{ArcSinh}\left[2\sqrt{\frac{3}{-5+\sqrt{73}}} x\right], \frac{1}{24}(-49+5\sqrt{73})\right]$$

Problem 131: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2+5x^2-7x^4}} dx$$

Optimal (type 4, 12 leaves, 2 steps):

$$\frac{\text{EllipticF}\left[\text{ArcSin}[x], -\frac{7}{2}\right]}{\sqrt{2}}$$

Result (type 4, 65 leaves):

$$\frac{i \sqrt{1-x^2} \sqrt{2+7x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{7}{2}} x\right], -\frac{2}{7}\right]}{\sqrt{7} \sqrt{2+5x^2-7x^4}}$$

Problem 132: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5x^2-8x^4}} dx$$

Optimal (type 4, 45 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{89}}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{4x}{\sqrt{5+\sqrt{89}}}\right], \frac{1}{32}(-57-5\sqrt{89})\right]$$

Result (type 4, 52 leaves):

$$-i \sqrt{\frac{2}{5+\sqrt{89}}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{4x}{\sqrt{-5+\sqrt{89}}}\right], \frac{1}{32}(-57+5\sqrt{89})\right]$$

Problem 133: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{2+5x^2-9x^4}} dx$$

Optimal (type 4, 49 leaves, 2 steps):

$$\sqrt{\frac{2}{-5+\sqrt{97}}} \text{EllipticF}\left[\text{ArcSin}\left[3\sqrt{\frac{2}{5+\sqrt{97}}}x\right], \frac{1}{36}(-61-5\sqrt{97})\right]$$

Result (type 4, 56 leaves):

$$-i \sqrt{\frac{2}{5+\sqrt{97}}} \text{EllipticF}\left[i \text{ArcSinh}\left[3\sqrt{\frac{2}{-5+\sqrt{97}}}x\right], \frac{1}{36}(-61+5\sqrt{97})\right]$$

Problem 171: Result more than twice size of optimal antiderivative.

$$\int \frac{(bx^2+cx^4)^3}{x^{15}} dx$$

Optimal (type 1, 19 leaves, 2 steps):

$$-\frac{(b+cx^2)^4}{8bx^8}$$

Result (type 1, 43 leaves):

$$-\frac{b^3}{8x^8} - \frac{b^2c}{2x^6} - \frac{3bc^2}{4x^4} - \frac{c^3}{2x^2}$$

Problem 352: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{7/2} \sqrt{b x^2 + c x^4} \, dx$$

Optimal (type 4, 323 leaves, 8 steps):

$$\frac{28 b^3 x^{3/2} (b + c x^2)}{195 c^{5/2} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{28 b^2 \sqrt{x} \sqrt{b x^2 + c x^4}}{585 c^2} + \frac{4 b x^{5/2} \sqrt{b x^2 + c x^4}}{117 c} + \frac{2}{13} x^{9/2} \sqrt{b x^2 + c x^4} - \left(28 b^{13/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) / \left(195 c^{11/4} \sqrt{b x^2 + c x^4} \right) + \left(14 b^{13/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) / \left(195 c^{11/4} \sqrt{b x^2 + c x^4} \right)$$

Result (type 4, 201 leaves):

$$\left(2 x^{3/2} \left(\sqrt{c} x \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (-14 b^3 - 4 b^2 c x^2 + 55 b c^2 x^4 + 45 c^3 x^6) + 42 b^{7/2} \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] - 42 b^{7/2} \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] \right) \right) / \left(585 c^{5/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 353: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{5/2} \sqrt{b x^2 + c x^4} \, dx$$

Optimal (type 4, 176 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{20 b^2 \sqrt{b x^2+c x^4}}{231 c^2 \sqrt{x}}+\frac{4 b x^{3 / 2} \sqrt{b x^2+c x^4}}{77 c}+\frac{2}{11} x^{7 / 2} \sqrt{b x^2+c x^4}+ \\
 & \left(10 b^{11 / 4} x(\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1 / 4} \sqrt{x}}{b^{1 / 4}}\right], \frac{1}{2}\right]\right) / \\
 & \left(231 c^{9 / 4} \sqrt{b x^2+c x^4}\right)
 \end{aligned}$$

Result (type 4, 133 leaves):

$$\begin{aligned}
 & \frac{1}{231} \sqrt{x^2(b+c x^2)} \\
 & \left(\frac{2(-10 b^2+6 b c x^2+21 c^2 x^4)}{c^2 \sqrt{x}}+\frac{20 i b^3 \sqrt{1+\frac{b}{c x^2}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^2(b+c x^2)}\right)
 \end{aligned}$$

Problem 354: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{3 / 2} \sqrt{b x^2+c x^4} d x$$

Optimal (type 4, 293 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{4 b^2 x^{3 / 2}(b+c x^2)}{15 c^{3 / 2}(\sqrt{b}+\sqrt{c} x) \sqrt{b x^2+c x^4}}+\frac{4 b \sqrt{x} \sqrt{b x^2+c x^4}}{45 c}+\frac{2}{9} x^{5 / 2} \sqrt{b x^2+c x^4}+ \\
 & \left(4 b^{9 / 4} x(\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1 / 4} \sqrt{x}}{b^{1 / 4}}\right], \frac{1}{2}\right]\right) / \\
 & \left(15 c^{7 / 4} \sqrt{b x^2+c x^4}\right)- \\
 & \left(2 b^{9 / 4} x(\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1 / 4} \sqrt{x}}{b^{1 / 4}}\right], \frac{1}{2}\right]\right) / \\
 & \left(15 c^{7 / 4} \sqrt{b x^2+c x^4}\right)
 \end{aligned}$$

Result (type 4, 190 leaves):

$$\left(2 x^{3/2} \left(\sqrt{c} x \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (2 b^2 + 7 b c x^2 + 5 c^2 x^4) - \right. \right. \\ \left. \left. 6 b^{5/2} \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] + \right. \right. \\ \left. \left. 6 b^{5/2} \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] \right) \right) / \\ \left(45 c^{3/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 355: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{x} \sqrt{b x^2 + c x^4} dx$$

Optimal (type 4, 146 leaves, 5 steps):

$$\frac{4 b \sqrt{b x^2 + c x^4}}{21 c \sqrt{x}} + \frac{2}{7} x^{3/2} \sqrt{b x^2 + c x^4} - \\ \left(2 b^{7/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}} \right], \frac{1}{2} \right] \right) / \\ (21 c^{5/4} \sqrt{b x^2 + c x^4})$$

Result (type 4, 120 leaves):

$$\frac{1}{21} \sqrt{x^2 (b + c x^2)} \left(\frac{4 b}{c \sqrt{x}} + 6 x^{3/2} - \frac{4 i b^2 \sqrt{1 + \frac{b}{c x^2}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \right], -1 \right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c (b + c x^2)} \right)$$

Problem 356: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b x^2 + c x^4}}{\sqrt{x}} dx$$

Optimal (type 4, 263 leaves, 6 steps):

$$\frac{4bx^{3/2}(b+cx^2)}{5\sqrt{c}(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} + \frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4} -$$

$$\left(4b^{5/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) /$$

$$\left(5c^{3/4}\sqrt{bx^2+cx^4} + \right.$$

$$\left. 2b^{5/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) /$$

$$\left(5c^{3/4}\sqrt{bx^2+cx^4} \right)$$

Result (type 4, 176 leaves):

$$\left(2x^{3/2} \right.$$

$$\left. \left(\sqrt{c}x\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}(b+cx^2) + 2b^{3/2}\sqrt{1+\frac{cx^2}{b}} \operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right], -1\right] - 2b^{3/2} \right. \right.$$

$$\left. \left. \sqrt{1+\frac{cx^2}{b}} \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right], -1\right] \right) \right) / \left(5\sqrt{c}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2(b+cx^2)} \right)$$

Problem 357: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}} dx$$

Optimal (type 4, 118 leaves, 4 steps):

$$\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}} + \left(2b^{3/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) /$$

$$\left(3c^{1/4}\sqrt{bx^2+cx^4} \right)$$

Result (type 4, 102 leaves):

$$\frac{2}{3} \sqrt{x^2 (b+c x^2)} \left(\frac{1}{\sqrt{x}} + \frac{2 i b \sqrt{1+\frac{b}{c x^2}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b+c x^2)} \right)$$

Problem 358: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b x^2+c x^4}}{x^{5/2}} dx$$

Optimal (type 4, 254 leaves, 6 steps):

$$\frac{4 \sqrt{c} x^{3/2} (b+c x^2)}{(\sqrt{b}+\sqrt{c} x) \sqrt{b x^2+c x^4}} - \frac{2 \sqrt{b x^2+c x^4}}{x^{3/2}} - \frac{1}{\sqrt{b x^2+c x^4}}$$

$$4 b^{1/4} c^{1/4} x (\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] +$$

$$\frac{1}{\sqrt{b x^2+c x^4}} 2 b^{1/4} c^{1/4} x (\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 175 leaves):

$$-\left(\left(2 \sqrt{x} \left(\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (b+c x^2) - \right. \right. \right. \\ \left. \left. \left. 2 \sqrt{b} \sqrt{c} x \sqrt{1+\frac{c x^2}{b}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] + 2 \sqrt{b} \sqrt{c} x \right. \right. \right. \\ \left. \left. \left. \sqrt{1+\frac{c x^2}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] \right) \right) / \left(\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b+c x^2)} \right) \right)$$

Problem 359: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b x^2+c x^4}}{x^{7/2}} dx$$

Optimal (type 4, 118 leaves, 4 steps):

$$-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}} + \left(2c^{3/4}x(\sqrt{b}+\sqrt{c}x) \sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) / \left(3b^{1/4}\sqrt{bx^2+cx^4} \right)$$

Result (type 4, 104 leaves):

$$\frac{2}{3}\sqrt{x^2(b+cx^2)} \left(-\frac{1}{x^{5/2}} + \frac{2ic\sqrt{1+\frac{b}{cx^2}} \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\right], -1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}(b+cx^2)} \right)$$

Problem 360: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}} dx$$

Optimal (type 4, 293 leaves, 7 steps):

$$\frac{4c^{3/2}x^{3/2}(b+cx^2)}{5b(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}} - \frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}} - \left(4c^{5/4}x(\sqrt{b}+\sqrt{c}x) \sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) / \left(5b^{3/4}\sqrt{bx^2+cx^4} \right) + \left(2c^{5/4}x(\sqrt{b}+\sqrt{c}x) \sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) / \left(5b^{3/4}\sqrt{bx^2+cx^4} \right)$$

Result (type 4, 196 leaves):

$$- \left(\left(2 \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (b^2 + 3 b c x^2 + 2 c^2 x^4) - 2 \sqrt{b} c^{3/2} x^3 \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] + 2 \sqrt{b} c^{3/2} x^3 \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] \right) / \left(5 b x^{3/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right) \right)$$

Problem 361: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b x^2 + c x^4}}{x^{11/2}} dx$$

Optimal (type 4, 146 leaves, 5 steps):

$$\frac{2 \sqrt{b x^2 + c x^4}}{7 x^{9/2}} - \frac{4 c \sqrt{b x^2 + c x^4}}{21 b x^{5/2}} - \left(2 c^{7/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}} \right], \frac{1}{2} \right] \right) / (21 b^{5/4} \sqrt{b x^2 + c x^4})$$

Result (type 4, 122 leaves):

$$\frac{1}{21} \sqrt{x^2 (b + c x^2)} \left(- \frac{2 (3 b + 2 c x^2)}{b x^{9/2}} - \frac{4 i c^2 \sqrt{1 + \frac{b}{c x^2}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right]}{b \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2)} \right)$$

Problem 362: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{b x^2 + c x^4}}{x^{13/2}} dx$$

Optimal (type 4, 323 leaves, 8 steps):

$$\begin{aligned}
 & -\frac{4c^{5/2}x^{3/2}(b+cx^2)}{15b^2(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}} - \frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}} + \frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}} + \\
 & \left(4c^{9/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) / \\
 & \left(15b^{7/4}\sqrt{bx^2+cx^4} \right) - \\
 & \left(2c^{9/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) / \\
 & \left(15b^{7/4}\sqrt{bx^2+cx^4} \right)
 \end{aligned}$$

Result (type 4, 209 leaves):

$$\begin{aligned}
 & 2 \left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}} (-5b^3 - 7b^2cx^2 + 4bc^2x^4 + 6c^3x^6) - \right. \\
 & 6\sqrt{b}c^{5/2}x^5\sqrt{1+\frac{cx^2}{b}} \operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right], -1\right] + \\
 & \left. 6\sqrt{b}c^{5/2}x^5\sqrt{1+\frac{cx^2}{b}} \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right], -1\right] \right) / \\
 & \left(45b^2x^{7/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2(b+cx^2)} \right)
 \end{aligned}$$

Problem 363: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}} dx$$

Optimal (type 4, 176 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}} - \frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}} + \frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}} + \\
 & \left(10c^{11/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) / \\
 & \left(231b^{9/4}\sqrt{bx^2+cx^4} \right)
 \end{aligned}$$

Result (type 4, 133 leaves):

$$\frac{1}{231} \sqrt{x^2 (b + c x^2)}$$

$$\left(\frac{2 (-21 b^2 - 6 b c x^2 + 10 c^2 x^4)}{b^2 x^{13/2}} + \frac{20 i c^3 \sqrt{1 + \frac{b}{c x^2}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{b^2 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (b + c x^2)} \right)$$

Problem 364: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{3/2} (b x^2 + c x^4)^{3/2} dx$$

Optimal (type 4, 350 leaves, 9 steps):

$$\frac{56 b^4 x^{3/2} (b + c x^2)}{1105 c^{5/2} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{56 b^3 \sqrt{x} \sqrt{b x^2 + c x^4}}{3315 c^2} +$$

$$\frac{8 b^2 x^{5/2} \sqrt{b x^2 + c x^4}}{663 c} + \frac{12}{221} b x^{9/2} \sqrt{b x^2 + c x^4} + \frac{2}{17} x^{5/2} (b x^2 + c x^4)^{3/2} -$$

$$\left(56 b^{17/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) /$$

$$\left(1105 c^{11/4} \sqrt{b x^2 + c x^4} \right) +$$

$$\left(28 b^{17/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) /$$

$$\left(1105 c^{11/4} \sqrt{b x^2 + c x^4} \right)$$

Result (type 4, 212 leaves):

$$\left(2 x^{3/2} \left(\sqrt{c} x \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (-28 b^4 - 8 b^3 c x^2 + 305 b^2 c^2 x^4 + 480 b c^3 x^6 + 195 c^4 x^8) + \right. \right. \\ \left. \left. 84 b^{9/2} \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] - \right. \right. \\ \left. \left. 84 b^{9/2} \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] \right) \right) / \\ \left(3315 c^{5/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 365: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{x} (b x^2 + c x^4)^{3/2} dx$$

Optimal (type 4, 203 leaves, 7 steps):

$$- \frac{8 b^3 \sqrt{b x^2 + c x^4}}{231 c^2 \sqrt{x}} + \frac{8 b^2 x^{3/2} \sqrt{b x^2 + c x^4}}{385 c} + \frac{4}{55} b x^{7/2} \sqrt{b x^2 + c x^4} + \frac{2}{15} x^{3/2} (b x^2 + c x^4)^{3/2} + \\ \left(4 b^{15/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}} \right], \frac{1}{2} \right] \right) / \\ \left(231 c^{9/4} \sqrt{b x^2 + c x^4} \right)$$

Result (type 4, 164 leaves):

$$\left(2 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} x^{3/2} (-20 b^4 - 8 b^3 c x^2 + 131 b^2 c^2 x^4 + 196 b c^3 x^6 + 77 c^4 x^8) + \right. \\ \left. 40 i b^4 \sqrt{1 + \frac{b}{c x^2}} x^2 \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \right], -1 \right] \right) / \left(1155 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^2 \sqrt{x^2 (b + c x^2)} \right)$$

Problem 366: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b x^2 + c x^4)^{3/2}}{\sqrt{x}} dx$$

Optimal (type 4, 320 leaves, 8 steps):

$$\begin{aligned} & -\frac{8 b^3 x^{3/2} (b+c x^2)}{65 c^{3/2} (\sqrt{b}+\sqrt{c} x) \sqrt{b x^2+c x^4}}+ \\ & \frac{8 b^2 \sqrt{x} \sqrt{b x^2+c x^4}}{195 c}+\frac{4}{39} b x^{5/2} \sqrt{b x^2+c x^4}+\frac{2}{13} \sqrt{x} (b x^2+c x^4)^{3/2}+ \\ & \left(8 b^{13/4} x(\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) / \\ & \left(65 c^{7/4} \sqrt{b x^2+c x^4}\right)- \\ & \left(4 b^{13/4} x(\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) / \\ & \left(65 c^{7/4} \sqrt{b x^2+c x^4}\right) \end{aligned}$$

Result (type 4, 201 leaves):

$$\begin{aligned} & \left(2 x^{3/2} \left(\sqrt{c} x \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\left(4 b^3+29 b^2 c x^2+40 b c^2 x^4+15 c^3 x^6\right)-\right.\right. \\ & \left.12 b^{7/2} \sqrt{1+\frac{c x^2}{b}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right],-1\right]+ \right. \\ & \left.12 b^{7/2} \sqrt{1+\frac{c x^2}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right],-1\right]\right) / \\ & \left(195 c^{3/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2(b+c x^2)}\right) \end{aligned}$$

Problem 367: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b x^2+c x^4)^{3/2}}{x^{3/2}} d x$$

Optimal (type 4, 173 leaves, 6 steps):

$$\frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}} + \frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{11\sqrt{x}} - \left(\frac{4b^{11/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{(77c^{5/4}\sqrt{bx^2+cx^4})} \right) /$$

Result (type 4, 153 leaves):

$$\left(2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}x^{3/2}(4b^3+17b^2cx^2+20bc^2x^4+7c^3x^6) - 8ib^3\sqrt{1+\frac{b}{cx^2}}x^2\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) / \left(77\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}c\sqrt{x^2(b+cx^2)} \right)$$

Problem 368: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}} dx$$

Optimal (type 4, 290 leaves, 7 steps):

$$\frac{8b^2x^{3/2}(b+cx^2)}{15\sqrt{c}(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} + \frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2(bx^2+cx^4)^{3/2}}{9x^{3/2}} - \left(\frac{8b^{9/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{(15c^{3/4}\sqrt{bx^2+cx^4})} + \frac{4b^{9/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{(15c^{3/4}\sqrt{bx^2+cx^4})} \right) /$$

Result (type 4, 190 leaves):

$$\left(2 x^{3/2} \left(\sqrt{c} x \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (11 b^2 + 16 b c x^2 + 5 c^2 x^4) + \right. \right. \\ \left. \left. 12 b^{5/2} \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] - \right. \right. \\ \left. \left. 12 b^{5/2} \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] \right) \right) / \\ \left(45 \sqrt{c} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 369: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b x^2 + c x^4)^{3/2}}{x^{7/2}} dx$$

Optimal (type 4, 143 leaves, 5 steps):

$$\frac{4 b \sqrt{b x^2 + c x^4}}{7 \sqrt{x}} + \frac{2 (b x^2 + c x^4)^{3/2}}{7 x^{5/2}} + \\ \left(4 b^{7/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}} \right], \frac{1}{2} \right] \right) / \\ \left(7 c^{1/4} \sqrt{b x^2 + c x^4} \right)$$

Result (type 4, 119 leaves):

$$\frac{1}{7 \sqrt{x^2 (b + c x^2)}} \\ 2 x^{3/2} \left(3 b^2 + 4 b c x^2 + c^2 x^4 + \frac{4 i b^2 \sqrt{1 + \frac{b}{c x^2}} \sqrt{x} \text{EllipticF} \left[i \text{ArcSinh} \left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}} \right)$$

Problem 370: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b x^2 + c x^4)^{3/2}}{x^{9/2}} dx$$

Optimal (type 4, 286 leaves, 7 steps):

$$\frac{24 b \sqrt{c} x^{3/2} (b + c x^2)}{5 (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} + \frac{12}{5} c \sqrt{x} \sqrt{b x^2 + c x^4} - \frac{2 (b x^2 + c x^4)^{3/2}}{x^{7/2}} - \frac{1}{5 \sqrt{b x^2 + c x^4}}$$

$$24 b^{5/4} c^{1/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] +$$

$$\frac{1}{5 \sqrt{b x^2 + c x^4}} 12 b^{5/4} c^{1/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 190 leaves):

$$\left(2 \sqrt{x} \left(\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (-5 b^2 - 4 b c x^2 + c^2 x^4) + \right. \right.$$

$$12 b^{3/2} \sqrt{c} x \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] - 12 b^{3/2} \sqrt{c} x$$

$$\left. \left. \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] \right) \right) / \left(5 \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 371: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b x^2 + c x^4)^{3/2}}{x^{11/2}} dx$$

Optimal (type 4, 143 leaves, 5 steps):

$$\frac{4 c \sqrt{b x^2 + c x^4}}{3 \sqrt{x}} - \frac{2 (b x^2 + c x^4)^{3/2}}{3 x^{9/2}} + \frac{1}{3 \sqrt{b x^2 + c x^4}}$$

$$4 b^{3/4} c^{3/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 111 leaves):

$$2 \left(-b^2 + c^2 x^4 + \frac{4 i b c \sqrt{1 + \frac{b}{c x^2}} x^{5/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{c} x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}} \right)$$

$$3 \sqrt{x} \sqrt{x^2 (b + c x^2)}$$

Problem 372: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b x^2 + c x^4)^{3/2}}{x^{13/2}} dx$$

Optimal (type 4, 287 leaves, 7 steps):

$$\frac{24 c^{3/2} x^{3/2} (b + c x^2)}{5 (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{12 c \sqrt{b x^2 + c x^4}}{5 x^{3/2}} - \frac{2 (b x^2 + c x^4)^{3/2}}{5 x^{11/2}} - \frac{1}{5 \sqrt{b x^2 + c x^4}}$$

$$24 b^{1/4} c^{5/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] +$$

$$\frac{1}{5 \sqrt{b x^2 + c x^4}} - 12 b^{1/4} c^{5/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]$$

Result (type 4, 193 leaves):

$$- \left(\left(2 \left(\sqrt{\frac{i \sqrt{c} x}}{\sqrt{b}} (b^2 + 8 b c x^2 + 7 c^2 x^4) - \right. \right. \right.$$

$$12 \sqrt{b} c^{3/2} x^3 \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}}{\sqrt{b}}}\right], -1\right] + 12 \sqrt{b} c^{3/2} x^3 \sqrt{1 + \frac{c x^2}{b}}$$

$$\left. \left. \left. \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}}{\sqrt{b}}}\right], -1\right] \right) \right) / \left(5 x^{3/2} \sqrt{\frac{i \sqrt{c} x}}{\sqrt{b}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 373: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b x^2 + c x^4)^{3/2}}{x^{15/2}} dx$$

Optimal (type 4, 143 leaves, 5 steps):

$$- \frac{4 c \sqrt{b x^2 + c x^4}}{7 x^{5/2}} - \frac{2 (b x^2 + c x^4)^{3/2}}{7 x^{13/2}} +$$

$$\left(4 c^{7/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) /$$

$$\left(7 b^{1/4} \sqrt{b x^2 + c x^4} \right)$$

Result (type 4, 120 leaves):

$$2 \left(\frac{-b^2 - 4bcx^2 - 3c^2x^4 + \frac{4ic^2 \sqrt{1 + \frac{b}{cx^2}} x^{9/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}}{7x^{5/2} \sqrt{x^2(b+cx^2)}} \right)$$

Problem 374: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{17/2}} dx$$

Optimal (type 4, 320 leaves, 8 steps):

$$\frac{8c^{5/2}x^{3/2}(b+cx^2)}{15b(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} - \frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}} - \frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{2(bx^2+cx^4)^{3/2}}{9x^{15/2}} - \left(\frac{8c^{9/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \text{EllipticE}\left[2\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{(15b^{3/4}\sqrt{bx^2+cx^4})} + \frac{4c^{9/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \text{EllipticF}\left[2\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{(15b^{3/4}\sqrt{bx^2+cx^4})} \right) /$$

Result (type 4, 209 leaves):

$$- \left(\left(2 \left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}} (5b^3 + 16b^2cx^2 + 23bc^2x^4 + 12c^3x^6) - 12\sqrt{b}c^{5/2}x^5 \sqrt{1 + \frac{cx^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right], -1\right] + 12\sqrt{b}c^{5/2}x^5 \sqrt{1 + \frac{cx^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right], -1\right] \right) \right) / \left(45bx^{7/2} \sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}} \sqrt{x^2(b+cx^2)} \right) \right)$$

Problem 375: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(bx^2 + cx^4)^{3/2}}{x^{19/2}} dx$$

Optimal (type 4, 173 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{12 c \sqrt{b x^2 + c x^4}}{77 x^{9/2}} - \frac{8 c^2 \sqrt{b x^2 + c x^4}}{77 b x^{5/2}} - \frac{2 (b x^2 + c x^4)^{3/2}}{11 x^{17/2}} \\
 & \left(4 c^{11/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) / \\
 & \left(77 b^{5/4} \sqrt{b x^2 + c x^4} \right)
 \end{aligned}$$

Result (type 4, 154 leaves):

$$\begin{aligned}
 & - \left(\left(2 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (7 b^3 + 20 b^2 c x^2 + 17 b c^2 x^4 + 4 c^3 x^6) + 4 i c^3 \sqrt{1 + \frac{b}{c x^2}} x^{13/2} \right. \right. \\
 & \left. \left. \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) \right) / \left(77 b \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} x^{9/2} \sqrt{x^2 (b + c x^2)} \right)
 \end{aligned}$$

Problem 376: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b x^2 + c x^4)^{3/2}}{x^{21/2}} dx$$

Optimal (type 4, 350 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{8 c^{7/2} x^{3/2} (b + c x^2)}{65 b^2 (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{4 c \sqrt{b x^2 + c x^4}}{39 x^{11/2}} \\
 & \frac{8 c^2 \sqrt{b x^2 + c x^4}}{195 b x^{7/2}} + \frac{8 c^3 \sqrt{b x^2 + c x^4}}{65 b^2 x^{3/2}} - \frac{2 (b x^2 + c x^4)^{3/2}}{13 x^{19/2}} + \\
 & \left(8 c^{13/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) / \\
 & \left(65 b^{7/4} \sqrt{b x^2 + c x^4} \right) - \\
 & \left(4 c^{13/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) / \\
 & \left(65 b^{7/4} \sqrt{b x^2 + c x^4} \right)
 \end{aligned}$$

Result (type 4, 220 leaves):

$$\begin{aligned}
 & \left(2 \left(\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \left(-15 b^4 - 40 b^3 c x^2 - 29 b^2 c^2 x^4 + 8 b c^3 x^6 + 12 c^4 x^8 \right) - \right. \right. \\
 & \quad 12 \sqrt{b} c^{7/2} x^7 \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] + \\
 & \quad \left. \left. 12 \sqrt{b} c^{7/2} x^7 \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] \right) \right) / \\
 & \left(195 b^2 x^{11/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)
 \end{aligned}$$

Problem 377: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(b x^2 + c x^4)^{3/2}}{x^{23/2}} dx$$

Optimal (type 4, 203 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{4 c \sqrt{b x^2 + c x^4}}{55 x^{13/2}} - \frac{8 c^2 \sqrt{b x^2 + c x^4}}{385 b x^{9/2}} + \frac{8 c^3 \sqrt{b x^2 + c x^4}}{231 b^2 x^{5/2}} - \frac{2 (b x^2 + c x^4)^{3/2}}{15 x^{21/2}} + \\
 & \left(4 c^{15/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}} \right], \frac{1}{2} \right] \right) / \\
 & \left(231 b^{9/4} \sqrt{b x^2 + c x^4} \right)
 \end{aligned}$$

Result (type 4, 165 leaves):

$$\begin{aligned}
 & \left(2 \left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \left(-77 b^4 - 196 b^3 c x^2 - 131 b^2 c^2 x^4 + 8 b c^3 x^6 + 20 c^4 x^8 \right) + \right. \right. \\
 & \quad \left. \left. 20 i c^4 \sqrt{1 + \frac{b}{c x^2}} x^{17/2} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right] \right) \right) / \\
 & \left(1155 b^2 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} x^{13/2} \sqrt{x^2 (b + c x^2)} \right)
 \end{aligned}$$

Problem 378: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}} dx$$

Optimal (type 4, 179 leaves, 6 steps):

$$\frac{30 b^2 \sqrt{bx^2+cx^4}}{77 c^3 \sqrt{x}} - \frac{18 b x^{3/2} \sqrt{bx^2+cx^4}}{77 c^2} + \frac{2 x^{7/2} \sqrt{bx^2+cx^4}}{11 c} -$$

$$\left(\frac{15 b^{11/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{77 c^{13/4} \sqrt{bx^2+cx^4}} \right) /$$

Result (type 4, 153 leaves):

$$\left(2 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} x^{3/2} (15 b^3 + 6 b^2 c x^2 - 2 b c^2 x^4 + 7 c^3 x^6) - \right.$$

$$\left. 30 i b^3 \sqrt{1 + \frac{b}{c x^2}} x^2 \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) / \left(77 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^3 \sqrt{x^2 (b+cx^2)} \right)$$

Problem 379: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}} dx$$

Optimal (type 4, 296 leaves, 7 steps):

$$\frac{14 b^2 x^{3/2} (b + c x^2)}{15 c^{5/2} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{14 b \sqrt{x} \sqrt{b x^2 + c x^4}}{45 c^2} + \frac{2 x^{5/2} \sqrt{b x^2 + c x^4}}{9 c} -$$

$$\left(14 b^{9/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) /$$

$$\left(15 c^{11/4} \sqrt{b x^2 + c x^4} \right) +$$

$$\left(7 b^{9/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) /$$

$$\left(15 c^{11/4} \sqrt{b x^2 + c x^4} \right)$$

Result (type 4, 190 leaves):

$$\left(2 x^{3/2} \left(\sqrt{c} x \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (-7 b^2 - 2 b c x^2 + 5 c^2 x^4) + \right. \right.$$

$$21 b^{5/2} \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] -$$

$$\left. \left. 21 b^{5/2} \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] \right) \right) /$$

$$\left(45 c^{5/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 380: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{9/2}}{\sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 149 leaves, 5 steps):

$$- \frac{10 b \sqrt{b x^2 + c x^4}}{21 c^2 \sqrt{x}} + \frac{2 x^{3/2} \sqrt{b x^2 + c x^4}}{7 c} +$$

$$\left(5 b^{7/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) /$$

$$\left(21 c^{9/4} \sqrt{b x^2 + c x^4} \right)$$

Result (type 4, 144 leaves):

$$\frac{x (b + c x^2) \left(-\frac{10 b \sqrt{x}}{21 c^2} + \frac{2 x^{5/2}}{7 c} \right)}{\sqrt{x^2 (b + c x^2)}} + \frac{10 i b^2 \sqrt{1 + \frac{b}{c x^2}} x^2 \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right]}{21 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^2 \sqrt{x^2 (b + c x^2)}}$$

Problem 381: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{7/2}}{\sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 266 leaves, 6 steps):

$$\frac{6 b x^{3/2} (b + c x^2)}{5 c^{3/2} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} + \frac{2 \sqrt{x} \sqrt{b x^2 + c x^4}}{5 c} + \left(\frac{6 b^{5/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}} \right], \frac{1}{2} \right]}{5 c^{7/4} \sqrt{b x^2 + c x^4}} - \frac{3 b^{5/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}} \right], \frac{1}{2} \right]}{5 c^{7/4} \sqrt{b x^2 + c x^4}} \right) /$$

Result (type 4, 176 leaves):

$$\left(2 x^{3/2} \left(\sqrt{c} x \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (b + c x^2) - 3 b^{3/2} \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] + 3 b^{3/2} \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] \right) \right) / \left(5 c^{3/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 382: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{5/2}}{\sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 121 leaves, 4 steps):

$$\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}} - \frac{b^{3/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \text{EllipticF}\left[2\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{3c^{5/4}\sqrt{bx^2+cx^4}}$$

Result (type 4, 126 leaves):

$$\frac{2x^{3/2}(b+cx^2)}{3c\sqrt{x^2(b+cx^2)}} - \frac{2\sqrt{b}\sqrt{1+\frac{b}{cx^2}}x^2 \text{EllipticF}\left[\text{i ArcSinh}\left[\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right], -1\right]}{3\sqrt{\frac{\text{i}\sqrt{b}}{\sqrt{c}}}\sqrt{cx^2(b+cx^2)}}$$

Problem 383: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}} dx$$

Optimal (type 4, 231 leaves, 5 steps):

$$\frac{2x^{3/2}(b+cx^2)}{\sqrt{c}(\sqrt{b}+\sqrt{c}x)\sqrt{bx^2+cx^4}} - \frac{1}{c^{3/4}\sqrt{bx^2+cx^4}}$$

$$2b^{1/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \text{EllipticE}\left[2\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] +$$

$$\frac{b^{1/4}x(\sqrt{b}+\sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b}+\sqrt{c}x)^2}} \text{EllipticF}\left[2\text{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{c^{3/4}\sqrt{bx^2+cx^4}}$$

Result (type 4, 112 leaves):

$$\left(2\sqrt{\text{i}x^{5/2}}\sqrt{1+\frac{cx^2}{b}}\right. \\ \left.\left(\text{EllipticE}\left[\text{i ArcSinh}\left[\sqrt{\frac{\text{i}\sqrt{c}x}{\sqrt{b}}}\right], -1\right] - \text{EllipticF}\left[\text{i ArcSinh}\left[\sqrt{\frac{\text{i}\sqrt{c}x}{\sqrt{b}}}\right], -1\right]\right)\right) / \\ \left(\left(\frac{\text{i}\sqrt{c}x}{\sqrt{b}}\right)^{3/2}\sqrt{x^2(b+cx^2)}\right)$$

Problem 384: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}} dx$$

Optimal (type 4, 90 leaves, 3 steps):

$$\frac{x \left(\sqrt{b} + \sqrt{c} x \right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b} + \sqrt{c} x \right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{b^{1/4} c^{1/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 85 leaves):

$$\frac{2 i \sqrt{1 + \frac{b}{c x^2}} x^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \sqrt{x^2 (b + c x^2)}}$$

Problem 385: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{x} \sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 259 leaves, 6 steps):

$$\frac{2 \sqrt{c} x^{3/2} (b + c x^2)}{b \left(\sqrt{b} + \sqrt{c} x \right) \sqrt{b x^2 + c x^4}} - \frac{2 \sqrt{b x^2 + c x^4}}{b x^{3/2}} - \frac{1}{b^{3/4} \sqrt{b x^2 + c x^4}}$$

$$+ 2 c^{1/4} x \left(\sqrt{b} + \sqrt{c} x \right) \sqrt{\frac{b + c x^2}{\left(\sqrt{b} + \sqrt{c} x \right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] +$$

$$\frac{c^{1/4} x \left(\sqrt{b} + \sqrt{c} x \right) \sqrt{\frac{b+c x^2}{\left(\sqrt{b} + \sqrt{c} x \right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{b^{3/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 177 leaves):

$$- \left(\left(2 \sqrt{x} \left(\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (b + c x^2) - \sqrt{b} \sqrt{c} x \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] + \right. \right. \right.$$

$$\left. \left. \sqrt{b} \sqrt{c} x \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] \right) \right) /$$

$$\left(b \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 386: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{3/2} \sqrt{bx^2 + cx^4}} dx$$

Optimal (type 4, 121 leaves, 4 steps):

$$-\frac{2\sqrt{bx^2 + cx^4}}{3bx^{5/2}} - \frac{c^{3/4}x(\sqrt{b} + \sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{3b^{5/4}\sqrt{bx^2 + cx^4}}$$

Result (type 4, 110 leaves):

$$2 \left(\frac{-b - cx^2 - \frac{ic\sqrt{1+\frac{b}{cx^2}}x^{5/2}\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}}{3b\sqrt{x}\sqrt{x^2(b+cx^2)}} \right)$$

Problem 387: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{5/2} \sqrt{bx^2 + cx^4}} dx$$

Optimal (type 4, 296 leaves, 7 steps):

$$-\frac{6c^{3/2}x^{3/2}(b+cx^2)}{5b^2(\sqrt{b} + \sqrt{c}x)\sqrt{bx^2 + cx^4}} - \frac{2\sqrt{bx^2 + cx^4}}{5bx^{7/2}} + \frac{6c\sqrt{bx^2 + cx^4}}{5b^2x^{3/2}} + \left(\frac{6c^{5/4}x(\sqrt{b} + \sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c}x)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5b^{7/4}\sqrt{bx^2 + cx^4}} - \frac{3c^{5/4}x(\sqrt{b} + \sqrt{c}x)\sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c}x)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5b^{7/4}\sqrt{bx^2 + cx^4}} \right) /$$

Result (type 4, 199 leaves):

$$\left(2 \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (-b^2 + 2 b c x^2 + 3 c^2 x^4) - \right. \\ \left. 6 \sqrt{b} c^{3/2} x^3 \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] + \right. \\ \left. 6 \sqrt{b} c^{3/2} x^3 \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] \right) / \\ \left(5 b^2 x^{3/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 388: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{7/2} \sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 149 leaves, 5 steps):

$$-\frac{2 \sqrt{b x^2 + c x^4}}{7 b x^{9/2}} + \frac{10 c \sqrt{b x^2 + c x^4}}{21 b^2 x^{5/2}} + \\ \left(5 c^{7/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}} \right], \frac{1}{2} \right] \right) / \\ \left(21 b^{9/4} \sqrt{b x^2 + c x^4} \right)$$

Result (type 4, 144 leaves):

$$\frac{\left(-\frac{2}{7 b x^{7/2}} + \frac{10 c}{21 b^2 x^{3/2}} \right) x (b + c x^2)}{\sqrt{x^2 (b + c x^2)}} + \frac{10 i c^2 \sqrt{1 + \frac{b}{c x^2}} x^2 \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}} \right], -1 \right]}{21 b^2 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \sqrt{x^2 (b + c x^2)}}$$

Problem 389: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{9/2} \sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 326 leaves, 8 steps):

$$\frac{14 c^{5/2} x^{3/2} (b + c x^2)}{15 b^3 (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{2 \sqrt{b x^2 + c x^4}}{9 b x^{11/2}} + \frac{14 c \sqrt{b x^2 + c x^4}}{45 b^2 x^{7/2}} - \frac{14 c^2 \sqrt{b x^2 + c x^4}}{15 b^3 x^{3/2}} -$$

$$\left(14 c^{9/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) /$$

$$\left(15 b^{11/4} \sqrt{b x^2 + c x^4} \right) +$$

$$\left(7 c^{9/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) /$$

$$\left(15 b^{11/4} \sqrt{b x^2 + c x^4} \right)$$

Result (type 4, 210 leaves):

$$\left(-2 \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (5 b^3 - 2 b^2 c x^2 + 14 b c^2 x^4 + 21 c^3 x^6) + \right.$$

$$42 \sqrt{b} c^{5/2} x^5 \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] -$$

$$42 \sqrt{b} c^{5/2} x^5 \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] \Bigg) /$$

$$\left(45 b^3 x^{7/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 390: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{11/2} \sqrt{b x^2 + c x^4}} dx$$

Optimal (type 4, 179 leaves, 6 steps):

$$-\frac{2 \sqrt{b x^2 + c x^4}}{11 b x^{13/2}} + \frac{18 c \sqrt{b x^2 + c x^4}}{77 b^2 x^{9/2}} - \frac{30 c^2 \sqrt{b x^2 + c x^4}}{77 b^3 x^{5/2}} -$$

$$\left(15 c^{11/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) /$$

$$\left(77 b^{13/4} \sqrt{b x^2 + c x^4} \right)$$

Result (type 4, 134 leaves):

$$\left(\left(2 \left(-7 b^3 + 2 b^2 c x^2 - 6 b c^2 x^4 - 15 c^3 x^6 - \frac{15 i c^3 \sqrt{1 + \frac{b}{c x^2}} x^{13/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}} \right) \right) \right) / \left(77 b^3 x^{9/2} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 391: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{17/2}}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 174 leaves, 6 steps):

$$-\frac{x^{11/2}}{c \sqrt{b x^2 + c x^4}} - \frac{15 b \sqrt{b x^2 + c x^4}}{7 c^3 \sqrt{x}} + \frac{9 x^{3/2} \sqrt{b x^2 + c x^4}}{7 c^2} + \left(\frac{15 b^{7/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{14 c^{13/4} \sqrt{b x^2 + c x^4}} \right) /$$

Result (type 4, 141 leaves):

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} x^{3/2} (-15 b^2 - 6 b c x^2 + 2 c^2 x^4) + 15 i b^2 \sqrt{1 + \frac{b}{c x^2}} x^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right] \right) / \left(7 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^3 \sqrt{x^2 (b + c x^2)} \right)$$

Problem 392: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{15/2}}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 291 leaves, 7 steps):

$$\begin{aligned} & -\frac{x^{9/2}}{c \sqrt{b x^2 + c x^4}} - \frac{21 b x^{3/2} (b + c x^2)}{5 c^{5/2} (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} + \frac{7 \sqrt{x} \sqrt{b x^2 + c x^4}}{5 c^2} + \\ & \left(\frac{21 b^{5/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 c^{11/4} \sqrt{b x^2 + c x^4}} - \right. \\ & \left. \frac{21 b^{5/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{10 c^{11/4} \sqrt{b x^2 + c x^4}} \right) / \end{aligned}$$

Result (type 4, 179 leaves):

$$\begin{aligned} & \left(x^{3/2} \left(\sqrt{c} x \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (7 b + 2 c x^2) - 21 b^{3/2} \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] + \right. \right. \\ & \left. \left. 21 b^{3/2} \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] \right) \right) / \\ & \left(5 c^{5/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right) \end{aligned}$$

Problem 393: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{13/2}}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 146 leaves, 5 steps):

$$-\frac{x^{7/2}}{c \sqrt{b x^2+c x^4}}+\frac{5 \sqrt{b x^2+c x^4}}{3 c^2 \sqrt{x}}-\left(5 b^{3/4} x(\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) / \left(6 c^{9/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 128 leaves):

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} x^{3/2}(5 b+2 c x^2)-5 i b \sqrt{1+\frac{b}{c x^2}} x^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right],-1\right]\right) / \left(3 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} c^2 \sqrt{x^2(b+c x^2)}\right)$$

Problem 394: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{11/2}}{(b x^2+c x^4)^{3/2}} dx$$

Optimal (type 4, 259 leaves, 6 steps):

$$-\frac{x^{5/2}}{c \sqrt{b x^2+c x^4}}+\frac{3 x^{3/2}(b+c x^2)}{c^{3/2}(\sqrt{b}+\sqrt{c} x) \sqrt{b x^2+c x^4}}-\frac{1}{c^{7/4} \sqrt{b x^2+c x^4}}+3 b^{1/4} x(\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]+ \left(3 b^{1/4} x(\sqrt{b}+\sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b}+\sqrt{c} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]\right) / \left(2 c^{7/4} \sqrt{b x^2+c x^4}\right)$$

Result (type 4, 167 leaves):

$$-\left(\left(x^{3/2}\left(\sqrt{c} x \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}-3 \sqrt{b} \sqrt{1+\frac{c x^2}{b}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right],-1\right]+3 \sqrt{b} \sqrt{1+\frac{c x^2}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right],-1\right]\right)\right) / \left(c^{3/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2(b+c x^2)}\right)$$

Problem 395: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{9/2}}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal (type 4, 119 leaves, 4 steps):

$$-\frac{x^{3/2}}{c\sqrt{bx^2 + cx^4}} + \frac{x(\sqrt{b} + \sqrt{c}x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c}x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{2b^{1/4}c^{5/4}\sqrt{bx^2 + cx^4}}$$

Result (type 4, 115 leaves):

$$\frac{-\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} x^{3/2} + i \sqrt{1 + \frac{b}{cx^2}} x^2 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\right], -1\right]}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}} c \sqrt{x^2(b + cx^2)}}$$

Problem 396: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{7/2}}{(bx^2 + cx^4)^{3/2}} dx$$

Optimal (type 4, 260 leaves, 6 steps):

$$\frac{x^{5/2}}{b\sqrt{bx^2 + cx^4}} - \frac{x^{3/2}(b + cx^2)}{b\sqrt{c}(\sqrt{b} + \sqrt{c}x)\sqrt{bx^2 + cx^4}} + \frac{x(\sqrt{b} + \sqrt{c}x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c}x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{b^{3/4}c^{3/4}\sqrt{bx^2 + cx^4}} - \frac{x(\sqrt{b} + \sqrt{c}x) \sqrt{\frac{b+cx^2}{(\sqrt{b} + \sqrt{c}x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4}\sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{2b^{3/4}c^{3/4}\sqrt{bx^2 + cx^4}}$$

Result (type 4, 168 leaves):

$$\left(i x^{5/2} \left(\sqrt{c} x \sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}} - \sqrt{b} \sqrt{1 + \frac{cx^2}{b}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right], -1\right] + \sqrt{b} \sqrt{1 + \frac{cx^2}{b}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right], -1\right] \right) \right) / \left(b^{3/2} \left(\frac{i\sqrt{c}x}{\sqrt{b}} \right)^{3/2} \sqrt{x^2(b + cx^2)} \right)$$

Problem 397: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{5/2}}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 118 leaves, 4 steps):

$$\frac{x^{3/2}}{b \sqrt{b x^2 + c x^4}} + \frac{x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b+c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{2 b^{5/4} c^{1/4} \sqrt{b x^2 + c x^4}}$$

Result (type 4, 114 leaves):

$$\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} x^{3/2} + i \sqrt{1 + \frac{b}{c x^2}} x^2 \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right], -1\right]}{b \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} \sqrt{x^2 (b + c x^2)}}$$

Problem 398: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{3/2}}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 286 leaves, 7 steps):

$$\frac{\sqrt{x}}{b \sqrt{b x^2 + c x^4}} + \frac{3 \sqrt{c} x^{3/2} (b + c x^2)}{b^2 (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{3 \sqrt{b x^2 + c x^4}}{b^2 x^{3/2}} - \frac{1}{b^{7/4} \sqrt{b x^2 + c x^4}}$$

$$+ \frac{3 c^{1/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{2}$$

$$\left(\frac{3 c^{1/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{2 b^{7/4} \sqrt{b x^2 + c x^4}} \right) /$$

Result (type 4, 181 leaves):

$$\begin{aligned}
 & - \left(\left(\sqrt{x} \left(\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (2b + 3c x^2) - 3 \sqrt{b} \sqrt{c} x \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticE} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] + \right. \right. \right. \\
 & \quad \left. \left. \left. 3 \sqrt{b} \sqrt{c} x \sqrt{1 + \frac{c x^2}{b}} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \right], -1 \right] \right) \right) / \right. \\
 & \quad \left. \left(b^2 \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right) \right)
 \end{aligned}$$

Problem 399: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{x}}{(b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 145 leaves, 5 steps):

$$\begin{aligned}
 & \frac{1}{b \sqrt{x} \sqrt{b x^2 + c x^4}} - \frac{5 \sqrt{b x^2 + c x^4}}{3 b^2 x^{5/2}} - \\
 & \left(5 c^{3/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \operatorname{EllipticF} \left[2 \operatorname{ArcTan} \left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}} \right], \frac{1}{2} \right] \right) / \\
 & \left(6 b^{9/4} \sqrt{b x^2 + c x^4} \right)
 \end{aligned}$$

Result (type 4, 110 leaves):

$$\begin{aligned}
 & -2b - 5c x^2 - \frac{5 i c \sqrt{1 + \frac{b}{c x^2}} x^{5/2} \operatorname{EllipticF} \left[i \operatorname{ArcSinh} \left[\sqrt{\frac{i \sqrt{b}}{\sqrt{c} x}} \right], -1 \right]}{\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}}} \\
 & \frac{\quad}{3 b^2 \sqrt{x} \sqrt{x^2 (b + c x^2)}}
 \end{aligned}$$

Problem 400: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{x} (b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 320 leaves, 8 steps):

$$\frac{1}{b x^{3/2} \sqrt{b x^2 + c x^4}} - \frac{21 c^{3/2} x^{3/2} (b + c x^2)}{5 b^3 (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \frac{7 \sqrt{b x^2 + c x^4}}{5 b^2 x^{7/2}} + \frac{21 c \sqrt{b x^2 + c x^4}}{5 b^3 x^{3/2}} +$$

$$\left(\frac{21 c^{5/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{5 b^{11/4} \sqrt{b x^2 + c x^4}} - \right.$$

$$\left. \frac{21 c^{5/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{10 b^{11/4} \sqrt{b x^2 + c x^4}} \right) /$$

Result (type 4, 198 leaves):

$$\left(\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} (-2 b^2 + 14 b c x^2 + 21 c^2 x^4) - \right.$$

$$21 \sqrt{b} c^{3/2} x^3 \sqrt{1 + \frac{c x^2}{b}} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] +$$

$$21 \sqrt{b} c^{3/2} x^3 \sqrt{1 + \frac{c x^2}{b}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}}\right], -1\right] \Big/$$

$$\left(5 b^3 x^{3/2} \sqrt{\frac{i \sqrt{c} x}{\sqrt{b}}} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 401: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{3/2} (b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 173 leaves, 6 steps):

$$\frac{1}{b x^{5/2} \sqrt{b x^2 + c x^4}} - \frac{9 \sqrt{b x^2 + c x^4}}{7 b^2 x^{9/2}} + \frac{15 c \sqrt{b x^2 + c x^4}}{7 b^3 x^{5/2}} +$$

$$\left(\frac{15 c^{7/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right]}{14 b^{13/4} \sqrt{b x^2 + c x^4}} \right) /$$

Result (type 4, 143 leaves):

$$\left(\sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} (-2 b^2 + 6 b c x^2 + 15 c^2 x^4) + \right. \\ \left. 15 i c^2 \sqrt{1 + \frac{b}{c x^2}} x^{9/2} \text{EllipticF}\left[\frac{i \sqrt{b}}{\sqrt{c}}, -1\right] \right) / \\ \left(7 b^3 \sqrt{\frac{i \sqrt{b}}{\sqrt{c}}} x^{5/2} \sqrt{x^2 (b + c x^2)} \right)$$

Problem 402: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{5/2} (b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 350 leaves, 9 steps):

$$\frac{1}{b x^{7/2} \sqrt{b x^2 + c x^4}} + \frac{77 c^{5/2} x^{3/2} (b + c x^2)}{15 b^4 (\sqrt{b} + \sqrt{c} x) \sqrt{b x^2 + c x^4}} - \\ \frac{11 \sqrt{b x^2 + c x^4}}{9 b^2 x^{11/2}} + \frac{77 c \sqrt{b x^2 + c x^4}}{45 b^3 x^{7/2}} - \frac{77 c^2 \sqrt{b x^2 + c x^4}}{15 b^4 x^{3/2}} - \\ \left(77 c^{9/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) / \\ \left(15 b^{15/4} \sqrt{b x^2 + c x^4} \right) + \\ \left(77 c^{9/4} x (\sqrt{b} + \sqrt{c} x) \sqrt{\frac{b + c x^2}{(\sqrt{b} + \sqrt{c} x)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} \sqrt{x}}{b^{1/4}}\right], \frac{1}{2}\right] \right) / \\ \left(30 b^{15/4} \sqrt{b x^2 + c x^4} \right)$$

Result (type 4, 210 leaves):

$$\left(-\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}} (10b^3 - 22b^2cx^2 + 154bc^2x^4 + 231c^3x^6) + \right. \\ \left. 231\sqrt{b}c^{5/2}x^5\sqrt{1+\frac{cx^2}{b}} \operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right], -1\right] - \right. \\ \left. 231\sqrt{b}c^{5/2}x^5\sqrt{1+\frac{cx^2}{b}} \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right], -1\right] \right) / \\ \left(45b^4x^{7/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2(b+cx^2)} \right)$$

Problem 407: Result more than twice size of optimal antiderivative.

$$\int \frac{(cx)^m}{(bx^2+cx^4)^2} dx$$

Optimal (type 5, 45 leaves, 3 steps):

$$\frac{(cx)^m \operatorname{Hypergeometric2F1}\left[2, \frac{1}{2}(-3+m), \frac{1}{2}(-1+m), -\frac{cx^2}{b}\right]}{b^2(3-m)x^3}$$

Result (type 5, 109 leaves):

$$\frac{1}{b^4x^3}(cx)^m \left(b \left(\frac{b}{-3+m} - \frac{2cx^2}{-1+m} \right) + \frac{2c^2x^4 \operatorname{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{cx^2}{b}\right]}{1+m} + \frac{c^2x^4 \operatorname{Hypergeometric2F1}\left[2, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{cx^2}{b}\right]}{1+m} \right)$$

Problem 408: Result more than twice size of optimal antiderivative.

$$\int \frac{(cx)^m}{(bx^2+cx^4)^3} dx$$

Optimal (type 5, 45 leaves, 3 steps):

$$\frac{(cx)^m \operatorname{Hypergeometric2F1}\left[3, \frac{1}{2}(-5+m), \frac{1}{2}(-3+m), -\frac{cx^2}{b}\right]}{b^3(5-m)x^5}$$

Result (type 5, 164 leaves):

$$\frac{1}{b^6 x^5} (c x)^m \left(\frac{b^3}{-5+m} - \frac{3 b^2 c x^2}{-3+m} + \frac{6 b c^2 x^4}{-1+m} - \frac{6 c^3 x^6 \operatorname{Hypergeometric2F1}\left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{c x^2}{b}\right]}{1+m} - \frac{3 c^3 x^6 \operatorname{Hypergeometric2F1}\left[2, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{c x^2}{b}\right]}{1+m} - \frac{c^3 x^6 \operatorname{Hypergeometric2F1}\left[3, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{c x^2}{b}\right]}{1+m} \right)$$

Problem 438: Result more than twice size of optimal antiderivative.

$$\int \frac{(a^2 + 2 a b x^2 + b^2 x^4)^2}{x^{11}} dx$$

Optimal (type 1, 19 leaves, 2 steps):

$$-\frac{(a + b x^2)^5}{10 a x^{10}}$$

Result (type 1, 52 leaves):

$$-\frac{a^4}{10 x^{10}} - \frac{a^3 b}{2 x^8} - \frac{a^2 b^2}{x^6} - \frac{a b^3}{x^4} - \frac{b^4}{2 x^2}$$

Problem 449: Result more than twice size of optimal antiderivative.

$$\int x^3 (a^2 + 2 a b x^2 + b^2 x^4)^3 dx$$

Optimal (type 1, 34 leaves, 4 steps):

$$-\frac{a (a + b x^2)^7}{14 b^2} + \frac{(a + b x^2)^8}{16 b^2}$$

Result (type 1, 77 leaves):

$$\frac{a^6 x^4}{4} + a^5 b x^6 + \frac{15}{8} a^4 b^2 x^8 + 2 a^3 b^3 x^{10} + \frac{5}{4} a^2 b^4 x^{12} + \frac{3}{7} a b^5 x^{14} + \frac{b^6 x^{16}}{16}$$

Problem 467: Result more than twice size of optimal antiderivative.

$$\int \frac{(a^2 + 2 a b x^2 + b^2 x^4)^3}{x^{15}} dx$$

Optimal (type 1, 19 leaves, 2 steps):

$$-\frac{(a + b x^2)^7}{14 a x^{14}}$$

Result (type 1, 82 leaves):

$$-\frac{a^6}{14 x^{14}} - \frac{a^5 b}{2 x^{12}} - \frac{3 a^4 b^2}{2 x^{10}} - \frac{5 a^3 b^3}{2 x^8} - \frac{5 a^2 b^4}{2 x^6} - \frac{3 a b^5}{2 x^4} - \frac{b^6}{2 x^2}$$

Problem 515: Result more than twice size of optimal antiderivative.

$$\int \frac{x^9}{(a^2 + 2 a b x^2 + b^2 x^4)^3} dx$$

Optimal (type 1, 19 leaves, 2 steps):

$$\frac{x^{10}}{10 a (a + b x^2)^5}$$

Result (type 1, 57 leaves):

$$-\frac{a^4 + 5 a^3 b x^2 + 10 a^2 b^2 x^4 + 10 a b^3 x^6 + 5 b^4 x^8}{10 b^5 (a + b x^2)^5}$$

Problem 659: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{(a^2 + 2 a b x^2 + b^2 x^4)^{1/3}} dx$$

Optimal (type 4, 298 leaves, 4 steps):

$$\frac{3 x (a + b x^2)}{5 b (a^2 + 2 a b x^2 + b^2 x^4)^{1/3}} +$$

$$\left(3 \times 3^{3/4} \sqrt{2 - \sqrt{3}} a^2 \left(1 + \frac{b x^2}{a}\right)^{2/3} \left(1 - \left(1 + \frac{b x^2}{a}\right)^{1/3}\right) \sqrt{\frac{1 + \left(1 + \frac{b x^2}{a}\right)^{1/3} + \left(1 + \frac{b x^2}{a}\right)^{2/3}}{\left(1 - \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}\right)^2}} \right)$$

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{1 + \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}}{1 - \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}}\right], -7 + 4 \sqrt{3}\right] /$$

$$\left(5 b^2 x (a^2 + 2 a b x^2 + b^2 x^4)^{1/3} \sqrt{\frac{1 - \left(1 + \frac{b x^2}{a}\right)^{1/3}}{\left(1 - \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}\right)^2}} \right)$$

Result (type 5, 64 leaves):

$$\frac{3 x \left(a + b x^2 - a \left(1 + \frac{b x^2}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, -\frac{b x^2}{a}\right]\right)}{5 b \left((a + b x^2)^2\right)^{1/3}}$$

Problem 660: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{1/3}} dx$$

Optimal (type 4, 256 leaves, 3 steps):

$$- \left(\left(3^{3/4} \sqrt{2-\sqrt{3}} a \left(1 + \frac{bx^2}{a} \right)^{2/3} \left(1 - \left(1 + \frac{bx^2}{a} \right)^{1/3} \right) \right. \right. \\ \left. \left. \sqrt{\frac{1 + \left(1 + \frac{bx^2}{a} \right)^{1/3} + \left(1 + \frac{bx^2}{a} \right)^{2/3}}{\left(1 - \sqrt{3} - \left(1 + \frac{bx^2}{a} \right)^{1/3} \right)^2}} \text{EllipticF} \left[\text{ArcSin} \left[\frac{1 + \sqrt{3} - \left(1 + \frac{bx^2}{a} \right)^{1/3}}{1 - \sqrt{3} - \left(1 + \frac{bx^2}{a} \right)^{1/3}} \right], -7 + 4\sqrt{3} \right] \right) / \right. \\ \left. \left(bx \left(a^2 + 2abx^2 + b^2x^4 \right)^{1/3} \sqrt{\frac{1 - \left(1 + \frac{bx^2}{a} \right)^{1/3}}{\left(1 - \sqrt{3} - \left(1 + \frac{bx^2}{a} \right)^{1/3} \right)^2}} \right) \right)$$

Result (type 5, 49 leaves):

$$\frac{x \left(\frac{a+bx^2}{a} \right)^{2/3} \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, -\frac{bx^2}{a} \right]}{\left((a+bx^2)^2 \right)^{1/3}}$$

Problem 661: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 (a^2 + 2abx^2 + b^2x^4)^{1/3}} dx$$

Optimal (type 4, 289 leaves, 4 steps):

$$\begin{aligned}
& - \frac{a + b x^2}{a x (a^2 + 2 a b x^2 + b^2 x^4)^{1/3}} + \\
& \left(\sqrt{2 - \sqrt{3}} \left(1 + \frac{b x^2}{a}\right)^{2/3} \left(1 - \left(1 + \frac{b x^2}{a}\right)^{1/3}\right) \sqrt{\frac{1 + \left(1 + \frac{b x^2}{a}\right)^{1/3} + \left(1 + \frac{b x^2}{a}\right)^{2/3}}{\left(1 - \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}\right)^2}} \right. \\
& \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1 + \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}}{1 - \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}}\right], -7 + 4 \sqrt{3}\right] \right) / \\
& \left(3^{1/4} x (a^2 + 2 a b x^2 + b^2 x^4)^{1/3} \sqrt{-\frac{1 - \left(1 + \frac{b x^2}{a}\right)^{1/3}}{\left(1 - \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}\right)^2}} \right)
\end{aligned}$$

Result (type 5, 72 leaves):

$$\frac{-3 (a + b x^2) - b x^2 \left(1 + \frac{b x^2}{a}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{3}{2}, -\frac{b x^2}{a}\right]}{3 a x \left((a + b x^2)^2\right)^{1/3}}$$

Problem 662: Result unnecessarily involves higher level functions.

$$\int \frac{x^2}{(a^2 + 2 a b x^2 + b^2 x^4)^{2/3}} dx$$

Optimal (type 4, 618 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{3 x (a+b x^2)}{2 b (a^2+2 a b x^2+b^2 x^4)^{2/3}} - \frac{9 a x \left(1+\frac{b x^2}{a}\right)^{4/3}}{2 b (a^2+2 a b x^2+b^2 x^4)^{2/3} \left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)} + \\
 & \left(9 \times 3^{1/4} \sqrt{2+\sqrt{3}} a^2 \left(1+\frac{b x^2}{a}\right)^{4/3} \left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}\right) \sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}} \right. \\
 & \quad \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}{1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}\right], -7+4 \sqrt{3}\right] \right) / \\
 & \left(4 b^2 x (a^2+2 a b x^2+b^2 x^4)^{2/3} \sqrt{-\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}} \right) - \\
 & \left(3 \times 3^{3/4} a^2 \left(1+\frac{b x^2}{a}\right)^{4/3} \left(1-\left(1+\frac{b x^2}{a}\right)^{1/3}\right) \sqrt{\frac{1+\left(1+\frac{b x^2}{a}\right)^{1/3}+\left(1+\frac{b x^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}} \right. \\
 & \quad \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}{1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}}\right], -7+4 \sqrt{3}\right] \right) / \\
 & \left(\sqrt{2} b^2 x (a^2+2 a b x^2+b^2 x^4)^{2/3} \sqrt{-\frac{1-\left(1+\frac{b x^2}{a}\right)^{1/3}}{\left(1-\sqrt{3}-\left(1+\frac{b x^2}{a}\right)^{1/3}\right)^2}} \right)
 \end{aligned}$$

Result (type 5, 64 leaves):

$$\frac{3 x (a+b x^2) \left(-1+\left(1+\frac{b x^2}{a}\right)^{1/3}\right) \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^2}{a}\right]}{2 b \left((a+b x^2)^2\right)^{2/3}}$$

Problem 663: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a^2+2 a b x^2+b^2 x^4)^{2/3}} dx$$

Optimal (type 4, 609 leaves, 6 steps):

$$\frac{3 x (a + b x^2)}{2 a (a^2 + 2 a b x^2 + b^2 x^4)^{2/3}} + \frac{3 x \left(1 + \frac{b x^2}{a}\right)^{4/3}}{2 (a^2 + 2 a b x^2 + b^2 x^4)^{2/3} \left(1 - \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}\right)}$$

$$\left(3 \times 3^{1/4} \sqrt{2 + \sqrt{3}} a \left(1 + \frac{b x^2}{a}\right)^{4/3} \left(1 - \left(1 + \frac{b x^2}{a}\right)^{1/3}\right) \sqrt{\frac{1 + \left(1 + \frac{b x^2}{a}\right)^{1/3} + \left(1 + \frac{b x^2}{a}\right)^{2/3}}{\left(1 - \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}\right)^2}} \right.$$

$$\left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{1 + \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}}{1 - \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}}\right], -7 + 4 \sqrt{3}\right] \right) /$$

$$\left(4 b x (a^2 + 2 a b x^2 + b^2 x^4)^{2/3} \sqrt{-\frac{1 - \left(1 + \frac{b x^2}{a}\right)^{1/3}}{\left(1 - \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}\right)^2}} \right. +$$

$$\left. 3^{3/4} a \left(1 + \frac{b x^2}{a}\right)^{4/3} \left(1 - \left(1 + \frac{b x^2}{a}\right)^{1/3}\right) \sqrt{\frac{1 + \left(1 + \frac{b x^2}{a}\right)^{1/3} + \left(1 + \frac{b x^2}{a}\right)^{2/3}}{\left(1 - \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}\right)^2}} \right.$$

$$\left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1 + \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}}{1 - \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}}\right], -7 + 4 \sqrt{3}\right] \right) /$$

$$\left(\sqrt{2} b x (a^2 + 2 a b x^2 + b^2 x^4)^{2/3} \sqrt{-\frac{1 - \left(1 + \frac{b x^2}{a}\right)^{1/3}}{\left(1 - \sqrt{3} - \left(1 + \frac{b x^2}{a}\right)^{1/3}\right)^2}} \right)$$

Result (type 5, 64 leaves):

$$\frac{x (a + b x^2) \left(-3 + \left(1 + \frac{b x^2}{a}\right)^{1/3}\right) \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{3}{2}, -\frac{b x^2}{a}\right]}{2 a \left((a + b x^2)^2\right)^{2/3}}$$

Problem 664: Result unnecessarily involves higher level functions.

$$\int \frac{1}{x^2 (a^2 + 2 a b x^2 + b^2 x^4)^{2/3}} dx$$

Optimal (type 4, 649 leaves, 7 steps):

$$\begin{aligned}
 & \frac{3(a+bx^2)}{2ax(a^2+2abx^2+b^2x^4)^{2/3}} - \frac{5(a+bx^2)^2}{2a^2x(a^2+2abx^2+b^2x^4)^{2/3}} \\
 & \frac{5bx\left(1+\frac{bx^2}{a}\right)^{4/3}}{2a(a^2+2abx^2+b^2x^4)^{2/3}\left(1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}\right)} + \\
 & \left(5 \times 3^{1/4} \sqrt{2+\sqrt{3}} \left(1+\frac{bx^2}{a}\right)^{4/3} \left(1-\left(1+\frac{bx^2}{a}\right)^{1/3}\right) \sqrt{\frac{1+\left(1+\frac{bx^2}{a}\right)^{1/3}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}\right)^2}} \right. \\
 & \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}}{1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}}\right], -7+4\sqrt{3}\right] \right) / \\
 & \left(4x(a^2+2abx^2+b^2x^4)^{2/3} \sqrt{-\frac{1-\left(1+\frac{bx^2}{a}\right)^{1/3}}{\left(1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}\right)^2}} \right) - \\
 & \left(5\left(1+\frac{bx^2}{a}\right)^{4/3} \left(1-\left(1+\frac{bx^2}{a}\right)^{1/3}\right) \sqrt{\frac{1+\left(1+\frac{bx^2}{a}\right)^{1/3}+\left(1+\frac{bx^2}{a}\right)^{2/3}}{\left(1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}\right)^2}} \right. \\
 & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1+\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}}{1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}}\right], -7+4\sqrt{3}\right] \right) / \\
 & \left(\sqrt{2} 3^{1/4} x (a^2+2abx^2+b^2x^4)^{2/3} \sqrt{-\frac{1-\left(1+\frac{bx^2}{a}\right)^{1/3}}{\left(1-\sqrt{3}-\left(1+\frac{bx^2}{a}\right)^{1/3}\right)^2}} \right)
 \end{aligned}$$

Result (type 5, 79 leaves):

$$\begin{aligned}
 & - \left(\left((a+bx^2) \left(6a+15bx^2-5bx^2 \left(1+\frac{bx^2}{a} \right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{3}{2}, -\frac{bx^2}{a}\right] \right) \right) \right) / \\
 & \left(6a^2x \left((a+bx^2)^2 \right)^{2/3} \right)
 \end{aligned}$$

Problem 909: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x(a+b+2ax^2+ax^4)} dx$$

Optimal (type 3, 69 leaves, 7 steps):

$$-\frac{\sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right]}{2\sqrt{b}(a+b)} + \frac{\operatorname{Log}[x]}{a+b} - \frac{\operatorname{Log}[a+b+2ax^2+ax^4]}{4(a+b)}$$

Result (type 3, 105 leaves):

$$\frac{1}{4\sqrt{b}(a+b)} \left(4\sqrt{b} \operatorname{Log}[x] + i \left(\sqrt{a} + i\sqrt{b} \right) \operatorname{Log}\left[-i\sqrt{b} + \sqrt{a}(1+x^2)\right] + \left(-i\sqrt{a} - \sqrt{b}\right) \operatorname{Log}\left[i\sqrt{b} + \sqrt{a}(1+x^2)\right] \right)$$

Problem 910: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^3(a+b+2ax^2+ax^4)} dx$$

Optimal (type 3, 89 leaves, 8 steps):

$$-\frac{1}{2(a+b)x^2} + \frac{\sqrt{a}(a-b) \operatorname{ArcTan}\left[\frac{\sqrt{a}(1+x^2)}{\sqrt{b}}\right]}{2\sqrt{b}(a+b)^2} - \frac{2a \operatorname{Log}[x]}{(a+b)^2} + \frac{a \operatorname{Log}[a+b+2ax^2+ax^4]}{2(a+b)^2}$$

Result (type 3, 163 leaves):

$$-\frac{1}{2(a+b)x^2} - \frac{2a \operatorname{Log}[x]}{(a+b)^2} + \frac{\left(-i a^2 + 2 a^{3/2} \sqrt{b} + i a b\right) \operatorname{Log}\left[\sqrt{a} - i \sqrt{b} + \sqrt{a} x^2\right]}{4 \sqrt{a} \sqrt{b} (a+b)^2} + \frac{\left(i a^2 + 2 a^{3/2} \sqrt{b} - i a b\right) \operatorname{Log}\left[\sqrt{a} + i \sqrt{b} + \sqrt{a} x^2\right]}{4 \sqrt{a} \sqrt{b} (a+b)^2}$$

Problem 911: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{a+b+2ax^2+ax^4} dx$$

Optimal (type 3, 432 leaves, 10 steps):

$$\frac{x}{a} + \frac{(a+b+2\sqrt{a}\sqrt{a+b}) \operatorname{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}a^{1/4}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} -$$

$$\frac{(a+b+2\sqrt{a}\sqrt{a+b}) \operatorname{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}a^{1/4}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a}+\sqrt{a+b}}} +$$

$$\left((a+b-2\sqrt{a}\sqrt{a+b}) \operatorname{Log}\left[\sqrt{a+b}-\sqrt{2}a^{1/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}x+\sqrt{a}x^2\right] \right) /$$

$$\left(4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}} \right) -$$

$$\left((a+b-2\sqrt{a}\sqrt{a+b}) \operatorname{Log}\left[\sqrt{a+b}+\sqrt{2}a^{1/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}x+\sqrt{a}x^2\right] \right) /$$

$$\left(4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{-\sqrt{a}+\sqrt{a+b}} \right)$$

Result (type 3, 164 leaves):

$$\frac{x}{a} - \frac{i(\sqrt{a}-i\sqrt{b})^2 \operatorname{ArcTan}\left[\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right]}{2a\sqrt{a-i\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{i(\sqrt{a}+i\sqrt{b})^2 \operatorname{ArcTan}\left[\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right]}{2a\sqrt{a+i\sqrt{a}\sqrt{b}}\sqrt{b}}$$

Problem 912: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{a+bx^2+ax^4} dx$$

Optimal (type 3, 331 leaves, 9 steps):

$$-\frac{\operatorname{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2}a^{1/4}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2}a^{1/4}x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a}+\sqrt{a+b}}} +$$

$$\frac{\operatorname{Log}\left[\sqrt{a+b}-\sqrt{2}a^{1/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}x+\sqrt{a}x^2\right]}{4\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}} -$$

$$\frac{\operatorname{Log}\left[\sqrt{a+b}+\sqrt{2}a^{1/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}x+\sqrt{a}x^2\right]}{4\sqrt{2}a^{3/4}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

Result (type 3, 143 leaves):

$$\frac{(i\sqrt{a}+\sqrt{b}) \operatorname{ArcTan}\left[\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right]}{\sqrt{a-i\sqrt{a}\sqrt{b}}} + \frac{(-i\sqrt{a}+\sqrt{b}) \operatorname{ArcTan}\left[\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right]}{\sqrt{a+i\sqrt{a}\sqrt{b}}}$$

$$2\sqrt{a}\sqrt{b}$$

Problem 913: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{a + b + 2 a x^2 + a x^4} dx$$

Optimal (type 3, 359 leaves, 9 steps):

$$\begin{aligned} & -\frac{\text{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2} a^{1/4} x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2 \sqrt{2} a^{1/4} \sqrt{a+b} \sqrt{\sqrt{a}+\sqrt{a+b}}} + \frac{\text{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2} a^{1/4} x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2 \sqrt{2} a^{1/4} \sqrt{a+b} \sqrt{\sqrt{a}+\sqrt{a+b}}} - \\ & \frac{\text{Log}\left[\sqrt{a+b}-\sqrt{2} a^{1/4} \sqrt{-\sqrt{a}+\sqrt{a+b}} x+\sqrt{a} x^2\right]}{4 \sqrt{2} a^{1/4} \sqrt{a+b} \sqrt{-\sqrt{a}+\sqrt{a+b}}} + \\ & \frac{\text{Log}\left[\sqrt{a+b}+\sqrt{2} a^{1/4} \sqrt{-\sqrt{a}+\sqrt{a+b}} x+\sqrt{a} x^2\right]}{4 \sqrt{2} a^{1/4} \sqrt{a+b} \sqrt{-\sqrt{a}+\sqrt{a+b}}} \end{aligned}$$

Result (type 3, 119 leaves):

$$\begin{aligned} & -\frac{i \text{ArcTan}\left[\frac{\sqrt{a} x}{\sqrt{a-i \sqrt{a} \sqrt{b}}}\right]}{2 \sqrt{a-i \sqrt{a} \sqrt{b}} \sqrt{b}} + \frac{i \text{ArcTan}\left[\frac{\sqrt{a} x}{\sqrt{a+i \sqrt{a} \sqrt{b}}}\right]}{2 \sqrt{a+i \sqrt{a} \sqrt{b}} \sqrt{b}} \end{aligned}$$

Problem 914: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 (a + b + 2 a x^2 + a x^4)} dx$$

Optimal (type 3, 433 leaves, 10 steps):

$$\begin{aligned} & -\frac{1}{(a+b) x} + \frac{a^{1/4} (2 \sqrt{a} + \sqrt{a+b}) \text{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}-\sqrt{2} a^{1/4} x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2 \sqrt{2} (a+b)^{3/2} \sqrt{\sqrt{a}+\sqrt{a+b}}} - \\ & \frac{a^{1/4} (2 \sqrt{a} + \sqrt{a+b}) \text{ArcTan}\left[\frac{\sqrt{-\sqrt{a}+\sqrt{a+b}}+\sqrt{2} a^{1/4} x}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right]}{2 \sqrt{2} (a+b)^{3/2} \sqrt{\sqrt{a}+\sqrt{a+b}}} + \\ & \left(a^{1/4} (2 \sqrt{a} - \sqrt{a+b}) \text{Log}\left[\sqrt{a+b}-\sqrt{2} a^{1/4} \sqrt{-\sqrt{a}+\sqrt{a+b}} x+\sqrt{a} x^2\right] \right) / \\ & \left(4 \sqrt{2} (a+b)^{3/2} \sqrt{-\sqrt{a}+\sqrt{a+b}} \right) - \\ & \left(a^{1/4} (2 \sqrt{a} - \sqrt{a+b}) \text{Log}\left[\sqrt{a+b}+\sqrt{2} a^{1/4} \sqrt{-\sqrt{a}+\sqrt{a+b}} x+\sqrt{a} x^2\right] \right) / \\ & \left(4 \sqrt{2} (a+b)^{3/2} \sqrt{-\sqrt{a}+\sqrt{a+b}} \right) \end{aligned}$$

Result (type 3, 174 leaves):

$$\frac{1}{(-a-b)x} + \frac{(i a - \sqrt{a} \sqrt{b}) \operatorname{ArcTan}\left[\frac{\sqrt{a} x}{\sqrt{a-i} \sqrt{a} \sqrt{b}}\right]}{2 \sqrt{a-i} \sqrt{a} \sqrt{b} \sqrt{b} (a+b)} + \frac{(-i a - \sqrt{a} \sqrt{b}) \operatorname{ArcTan}\left[\frac{\sqrt{a} x}{\sqrt{a+i} \sqrt{a} \sqrt{b}}\right]}{2 \sqrt{a+i} \sqrt{a} \sqrt{b} \sqrt{b} (a+b)}$$

Problem 918: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{1-x^2+x^4} dx$$

Optimal (type 3, 74 leaves, 9 steps):

$$-\frac{1}{2} \operatorname{ArcTan}[\sqrt{3}-2x] + \frac{1}{2} \operatorname{ArcTan}[\sqrt{3}+2x] + \frac{\operatorname{Log}[1-\sqrt{3}x+x^2]}{4\sqrt{3}} - \frac{\operatorname{Log}[1+\sqrt{3}x+x^2]}{4\sqrt{3}}$$

Result (type 3, 94 leaves):

$$\frac{1}{2\sqrt{6}} \left(\sqrt{-1-i\sqrt{3}} (i+\sqrt{3}) \operatorname{ArcTan}\left[\frac{1}{2}(1-i\sqrt{3})x\right] + \sqrt{-1+i\sqrt{3}} (-i+\sqrt{3}) \operatorname{ArcTan}\left[\frac{1}{2}(1+i\sqrt{3})x\right] \right)$$

Problem 919: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{2-2x^2+x^4} dx$$

Optimal (type 3, 188 leaves, 9 steps):

$$-\frac{1}{2} \sqrt{\frac{1}{2}(1+\sqrt{2})} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}-2x}{\sqrt{2(-1+\sqrt{2})}}\right] + \frac{1}{2} \sqrt{\frac{1}{2}(1+\sqrt{2})} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}+2x}{\sqrt{2(-1+\sqrt{2})}}\right] + \frac{\operatorname{Log}[\sqrt{2}-\sqrt{2(1+\sqrt{2})}x+x^2]}{4\sqrt{2(1+\sqrt{2})}} - \frac{\operatorname{Log}[\sqrt{2}+\sqrt{2(1+\sqrt{2})}x+x^2]}{4\sqrt{2(1+\sqrt{2})}}$$

Result (type 3, 39 leaves):

$$-\frac{\operatorname{ArcTan}\left[\frac{x}{\sqrt{-1-i}}\right]}{(-1-i)^{3/2}} - \frac{\operatorname{ArcTan}\left[\frac{x}{\sqrt{-1+i}}\right]}{(-1+i)^{3/2}}$$

Problem 930: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 \sqrt{a + b x^2 + c x^4} dx$$

Optimal (type 4, 395 leaves, 5 steps):

$$\begin{aligned} & -\frac{2(2b^2 - 5ac)x\sqrt{a + bx^2 + cx^4}}{105c^2} + \frac{b(8b^2 - 29ac)x\sqrt{a + bx^2 + cx^4}}{105c^{5/2}(\sqrt{a} + \sqrt{c}x^2)} + \\ & \frac{x^3(b + 5cx^2)\sqrt{a + bx^2 + cx^4}}{35c} - \left(a^{1/4}b(8b^2 - 29ac)(\sqrt{a} + \sqrt{c}x^2) \sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{c}x^2)^2}} \right. \\ & \left. \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \left(105c^{11/4}\sqrt{a + bx^2 + cx^4}\right) + \\ & \left(a^{1/4}(8b^3 - 29abc + 2\sqrt{a}\sqrt{c}(2b^2 - 5ac))(\sqrt{a} + \sqrt{c}x^2) \sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{c}x^2)^2}} \right. \\ & \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \left(210c^{11/4}\sqrt{a + bx^2 + cx^4}\right) \end{aligned}$$

Result (type 4, 538 leaves):

$$\begin{aligned} & \frac{1}{420c^3\sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}}\sqrt{a + bx^2 + cx^4}} \left(4c\sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}}x \right. \\ & \left. (10a^2c - 4b^3x^2 - b^2cx^4 + 18b^2c^2x^6 + 15c^3x^8 + a(-4b^2 + 13bcx^2 + 25c^2x^4)) + \right. \\ & \left. ib(8b^2 - 29ac)(-b + \sqrt{b^2 - 4ac})\sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{b + \sqrt{b^2 - 4ac}}}\sqrt{\frac{2b - 2\sqrt{b^2 - 4ac} + 4cx^2}{b - \sqrt{b^2 - 4ac}}} \right. \\ & \left. \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}}x\right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}}\right] - \right. \\ & \left. i(-8b^4 + 37ab^2c - 20a^2c^2 + 8b^3\sqrt{b^2 - 4ac} - 29abc\sqrt{b^2 - 4ac})\sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{b + \sqrt{b^2 - 4ac}}} \right. \\ & \left. \sqrt{\frac{2b - 2\sqrt{b^2 - 4ac} + 4cx^2}{b - \sqrt{b^2 - 4ac}}}\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}}x\right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}}\right] \right) \end{aligned}$$

Problem 931: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 \sqrt{a+bx^2+cx^4} dx$$

Optimal (type 4, 342 leaves, 4 steps):

$$\begin{aligned} & -\frac{2(b^2-3ac)x\sqrt{a+bx^2+cx^4}}{15c^{3/2}(\sqrt{a}+\sqrt{c}x^2)} + \frac{x(b+3cx^2)\sqrt{a+bx^2+cx^4}}{15c} + \\ & \left(2a^{1/4}(b^2-3ac)(\sqrt{a}+\sqrt{c}x^2) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}} \right. \\ & \left. \text{EllipticE}\left[2\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \left(15c^{7/4}\sqrt{a+bx^2+cx^4}\right) - \\ & \left(a^{1/4}(2b^2+\sqrt{a}b\sqrt{c}-6ac)(\sqrt{a}+\sqrt{c}x^2) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}} \right. \\ & \left. \text{EllipticF}\left[2\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \left(30c^{7/4}\sqrt{a+bx^2+cx^4}\right) \end{aligned}$$

Result (type 4, 479 leaves):

$$\begin{aligned} & \frac{1}{30c^2 \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} \sqrt{a+bx^2+cx^4}} \left(2c \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} x(b+3cx^2)(a+bx^2+cx^4) - \right. \\ & \left. i(b^2-3ac)(-b+\sqrt{b^2-4ac}) \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}} \sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}} \right. \\ & \left. \text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] + \right. \\ & \left. i(-b^3+4abc+b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}) \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}} \right. \\ & \left. \sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}} \text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] \right) \end{aligned}$$

Problem 932: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + b x^2 + c x^4} \, dx$$

Optimal (type 4, 309 leaves, 4 steps):

$$\frac{1}{3} x \sqrt{a + b x^2 + c x^4} + \frac{b x \sqrt{a + b x^2 + c x^4}}{3 \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} - \left(a^{1/4} b (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right) / \left(3 c^{3/4} \sqrt{a + b x^2 + c x^4} \right) + \left(a^{1/4} (b + 2 \sqrt{a} \sqrt{c}) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right) / \left(6 c^{3/4} \sqrt{a + b x^2 + c x^4} \right)$$

Result (type 4, 445 leaves):

$$\left(4 c \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x (a + b x^2 + c x^4) + i b (-b + \sqrt{b^2 - 4 a c}) \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right] - i (-b^2 + 4 a c + b \sqrt{b^2 - 4 a c}) \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right] \right) / \left(12 c \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \sqrt{a + b x^2 + c x^4} \right)$$

Problem 933: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^2} dx$$

Optimal (type 4, 303 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+bx^2+cx^4}}{x} + \frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2} - \frac{1}{\sqrt{a+bx^2+cx^4}} \\
 & 2a^{1/4}c^{1/4}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}} \text{EllipticE}\left[2\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] + \\
 & \left((b+2\sqrt{a}\sqrt{c})(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}} \right. \\
 & \left. \text{EllipticF}\left[2\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \left(2a^{1/4}c^{1/4}\sqrt{a+bx^2+cx^4}\right)
 \end{aligned}$$

Result (type 4, 435 leaves):

$$\begin{aligned}
 & \left(-2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}(a+bx^2+cx^4) + \right. \\
 & i(-b+\sqrt{b^2-4ac})x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}} \\
 & \text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right]x, \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] - \\
 & i\sqrt{2}\sqrt{b^2-4ac}x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}} \\
 & \left. \text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right]x, \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] \right) / \\
 & \left(2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4} \right)
 \end{aligned}$$

Problem 934: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + b x^2 + c x^4}}{x^4} dx$$

Optimal (type 4, 341 leaves, 5 steps):

$$\begin{aligned} & -\frac{\sqrt{a + b x^2 + c x^4}}{3 x^3} - \frac{b \sqrt{a + b x^2 + c x^4}}{3 a x} + \frac{b \sqrt{c} x \sqrt{a + b x^2 + c x^4}}{3 a (\sqrt{a} + \sqrt{c} x^2)} - \\ & \left(b c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right) / \\ & \left(3 a^{3/4} \sqrt{a + b x^2 + c x^4} \right) + \left((b + 2 \sqrt{a} \sqrt{c}) c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right. \\ & \left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right) / \left(6 a^{3/4} \sqrt{a + b x^2 + c x^4} \right) \end{aligned}$$

Result (type 4, 459 leaves):

$$\begin{aligned} & \frac{1}{12 a \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x^3 \sqrt{a + b x^2 + c x^4}} \left(-4 \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} (a + b x^2) (a + b x^2 + c x^4) + \right. \\ & i b \left(-b + \sqrt{b^2 - 4 a c} \right) x^3 \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \\ & \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] - \\ & i \left(-b^2 + 4 a c + b \sqrt{b^2 - 4 a c} \right) x^3 \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \\ & \left. \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] \right) \end{aligned}$$

Problem 935: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + b x^2 + c x^4}}{x^6} dx$$

Optimal (type 4, 397 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+bx^2+cx^4}}{5x^5} - \frac{b\sqrt{a+bx^2+cx^4}}{15ax^3} + \frac{2(b^2-3ac)\sqrt{a+bx^2+cx^4}}{15a^2x} - \\
 & \frac{2\sqrt{c}(b^2-3ac)x\sqrt{a+bx^2+cx^4}}{15a^2(\sqrt{a}+\sqrt{c}x^2)} + \left(2c^{1/4}(b^2-3ac)(\sqrt{a}+\sqrt{c}x^2) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}} \right. \\
 & \left. \text{EllipticE}\left[2\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \left(15a^{7/4}\sqrt{a+bx^2+cx^4} \right) - \\
 & \left(c^{1/4}(2b^2+\sqrt{a}b\sqrt{c}-6ac)(\sqrt{a}+\sqrt{c}x^2) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}} \right. \\
 & \left. \text{EllipticF}\left[2\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \left(30a^{7/4}\sqrt{a+bx^2+cx^4} \right)
 \end{aligned}$$

Result (type 4, 530 leaves):

$$\begin{aligned}
 & \frac{1}{30a^2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} x^5 \sqrt{a+bx^2+cx^4} \\
 & \left(-2 \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} (3a^3-2b^2x^6(b+cx^2)+a^2(4bx^2+9cx^4)+a(-b^2x^4+7bcx^6+6c^2x^8)) - \right. \\
 & \quad \left. i(b^2-3ac)(-b+\sqrt{b^2-4ac})x^5 \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}} \sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}} \right. \\
 & \quad \left. \text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] + \right. \\
 & \quad \left. i(-b^3+4abc+b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac})x^5 \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}} \right. \\
 & \quad \left. \sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}} \text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] \right)
 \end{aligned}$$

Problem 947: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 (a+bx^2+cx^4)^{3/2} dx$$

Optimal (type 4, 495 leaves, 6 steps):

$$\frac{(8 b^4 - 51 a b^2 c + 60 a^2 c^2) x \sqrt{a + b x^2 + c x^4}}{1155 c^3} - \frac{8 b (2 b^2 - 9 a c) (b^2 - 3 a c) x \sqrt{a + b x^2 + c x^4}}{1155 c^{7/2} (\sqrt{a} + \sqrt{c} x^2)} -$$

$$\frac{x^3 (b (2 b^2 + a c) + 10 c (b^2 - 3 a c) x^2) \sqrt{a + b x^2 + c x^4}}{385 c^2} + \frac{x^3 (b + 3 c x^2) (a + b x^2 + c x^4)^{3/2}}{33 c} +$$

$$\left(8 a^{1/4} b (2 b^2 - 9 a c) (b^2 - 3 a c) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right.$$

$$\left. \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(1155 c^{15/4} \sqrt{a + b x^2 + c x^4} \right) -$$

$$\left(a^{1/4} (8 b (2 b^2 - 9 a c) (b^2 - 3 a c) + \sqrt{a} \sqrt{c} (8 b^4 - 51 a b^2 c + 60 a^2 c^2)) \right.$$

$$\left. (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right.$$

$$\left. \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(2310 c^{15/4} \sqrt{a + b x^2 + c x^4} \right)$$

Result (type 4, 657 leaves):

$$\begin{aligned}
 & \frac{1}{2310 c^4 \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} \sqrt{a+b x^2+c x^4}} \\
 & \left(2 c \sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} x \left(60 a^3 c^2 + a^2 c \left(-51 b^2 + 92 b c x^2 + 255 c^2 x^4 \right) + \right. \right. \\
 & \quad \left. \left. a \left(8 b^4 - 57 b^3 c x^2 - 14 b^2 c^2 x^4 + 367 b c^3 x^6 + 300 c^4 x^8 \right) + \right. \right. \\
 & \quad \left. \left. x^2 \left(8 b^5 + 2 b^4 c x^2 - b^3 c^2 x^4 + 145 b^2 c^3 x^6 + 245 b c^4 x^8 + 105 c^5 x^{10} \right) \right) - \right. \\
 & \quad \left. 4 i b \left(2 b^4 - 15 a b^2 c + 27 a^2 c^2 \right) \left(-b + \sqrt{b^2 - 4 a c} \right) \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \right. \\
 & \quad \left. \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \right. \\
 & \quad \left. \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. i \left(-8 b^6 + 68 a b^4 c - 159 a^2 b^2 c^2 + 60 a^3 c^3 + 8 b^5 \sqrt{b^2 - 4 a c} - 60 a b^3 c \sqrt{b^2 - 4 a c} + \right. \right. \\
 & \quad \left. \left. 108 a^2 b c^2 \sqrt{b^2 - 4 a c} \right) \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \right. \\
 & \quad \left. \left. \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] \right) \right)
 \end{aligned}$$

Problem 948: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2 (a + b x^2 + c x^4)^{3/2} dx$$

Optimal (type 4, 443 leaves, 5 steps):

$$\frac{(8 b^4 - 57 a b^2 c + 84 a^2 c^2) x \sqrt{a + b x^2 + c x^4}}{315 c^{5/2} (\sqrt{a} + \sqrt{c} x^2)} -$$

$$\frac{x (b (4 b^2 - 9 a c) + 6 c (2 b^2 - 7 a c) x^2) \sqrt{a + b x^2 + c x^4}}{315 c^2} + \frac{x (3 b + 7 c x^2) (a + b x^2 + c x^4)^{3/2}}{63 c} -$$

$$\left(a^{1/4} (8 b^4 - 57 a b^2 c + 84 a^2 c^2) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right.$$

$$\left. \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(315 c^{11/4} \sqrt{a + b x^2 + c x^4} \right) +$$

$$\left(a^{1/4} (8 b^4 - 57 a b^2 c + 84 a^2 c^2 + 4 \sqrt{a} b \sqrt{c} (b^2 - 6 a c)) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right.$$

$$\left. \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(630 c^{11/4} \sqrt{a + b x^2 + c x^4} \right)$$

Result (type 4, 602 leaves):

$$\frac{1}{1260 c^3 \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \sqrt{a + b x^2 + c x^4}}$$

$$\left(4 c \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x (-4 b^4 x^2 - b^3 c x^4 + 53 b^2 c^2 x^6 + 85 b c^3 x^8 + 35 c^4 x^{10} + a^2 c (24 b + 77 c x^2) + \right.$$

$$\left. a (-4 b^3 + 27 b^2 c x^2 + 151 b c^2 x^4 + 112 c^3 x^6) \right) + i (8 b^4 - 57 a b^2 c + 84 a^2 c^2)$$

$$\left(-b + \sqrt{b^2 - 4 a c} \right) \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}}$$

$$\text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] -$$

$$i \left(-8 b^5 + 65 a b^3 c - 132 a^2 b c^2 + 8 b^4 \sqrt{b^2 - 4 a c} - 57 a b^2 c \sqrt{b^2 - 4 a c} + 84 a^2 c^2 \sqrt{b^2 - 4 a c} \right)$$

$$\sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}}$$

$$\left. \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] \right)$$

Problem 949: Result unnecessarily involves imaginary or complex numbers.

$$\int (a+b x^2+c x^4)^{3/2} dx$$

Optimal (type 4, 381 leaves, 5 steps):

$$\begin{aligned} & -\frac{2 b\left(b^2-8 a c\right) x \sqrt{a+b x^2+c x^4}}{35 c^{3/2}\left(\sqrt{a}+\sqrt{c} x^2\right)}+\frac{x\left(b^2+10 a c+3 b c x^2\right) \sqrt{a+b x^2+c x^4}}{35 c}+ \\ & \frac{1}{7} x\left(a+b x^2+c x^4\right)^{3/2}+\left(2 a^{1/4} b\left(b^2-8 a c\right)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\right. \\ & \left.\text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) / \left(35 c^{7/4} \sqrt{a+b x^2+c x^4}\right)- \\ & \left(a^{1/4}\left(\sqrt{a}-\sqrt{c}\left(b^2-20 a c\right)+2 b\left(b^2-8 a c\right)\right)\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\right. \\ & \left.\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) / \left(70 c^{7/4} \sqrt{a+b x^2+c x^4}\right) \end{aligned}$$

Result (type 4, 533 leaves):

$$\begin{aligned} & \frac{1}{70 c^2 \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}} \sqrt{a+b x^2+c x^4}}}\left(2 c \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right. \\ & \left.(15 a^2 c+a\left(b^2+23 b c x^2+20 c^2 x^4\right)+x^2\left(b^3+9 b^2 c x^2+13 b c^2 x^4+5 c^3 x^6\right)\right)- \\ & i b\left(b^2-8 a c\right)\left(-b+\sqrt{b^2-4 a c}\right) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}} \\ & \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]+ \\ & i\left(-b^4+9 a b^2 c-20 a^2 c^2+b^3 \sqrt{b^2-4 a c}-8 a b c \sqrt{b^2-4 a c}\right) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \\ & \left.\sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right) \end{aligned}$$

Problem 950: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b x^2 + c x^4)^{3/2}}{x^2} dx$$

Optimal (type 4, 361 leaves, 5 steps):

$$\frac{(b^2 + 12 a c) x \sqrt{a + b x^2 + c x^4}}{5 \sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} + \frac{1}{5} x (7 b + 6 c x^2) \sqrt{a + b x^2 + c x^4} -$$

$$\frac{(a + b x^2 + c x^4)^{3/2}}{x} - \left(a^{1/4} (b^2 + 12 a c) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right.$$

$$\left. \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(5 c^{3/4} \sqrt{a + b x^2 + c x^4} \right) +$$

$$\left(a^{1/4} (b^2 + 8 \sqrt{a} b \sqrt{c} + 12 a c) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right.$$

$$\left. \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(10 c^{3/4} \sqrt{a + b x^2 + c x^4} \right)$$

Result (type 4, 505 leaves):

$$\frac{1}{20 c \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \sqrt{a + b x^2 + c x^4}}$$

$$\left(4 c \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} (-5 a^2 - 3 a b x^2 + 2 b^2 x^4 - 4 a c x^4 + 3 b c x^6 + c^2 x^8) + \right.$$

$$i (b^2 + 12 a c) (-b + \sqrt{b^2 - 4 a c}) x \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}}$$

$$\left. \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] - \right.$$

$$i \left(-b^3 + 4 a b c + b^2 \sqrt{b^2 - 4 a c} + 12 a c \sqrt{b^2 - 4 a c} \right) x \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}}$$

$$\left. \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] \right)$$

Problem 951: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b x^2 + c x^4)^{3/2}}{x^4} dx$$

Optimal (type 4, 353 leaves, 5 steps):

$$\frac{8 b \sqrt{c} x \sqrt{a + b x^2 + c x^4}}{3 (\sqrt{a} + \sqrt{c} x^2)} - \frac{(3 b - 2 c x^2) \sqrt{a + b x^2 + c x^4}}{3 x} - \frac{(a + b x^2 + c x^4)^{3/2}}{3 x^3} - \frac{1}{3 \sqrt{a + b x^2 + c x^4}}$$

$$8 a^{1/4} b c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] +$$

$$\left((3 b^2 + 8 \sqrt{a} b \sqrt{c} + 4 a c) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right.$$

$$\left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right) / \left(6 a^{1/4} c^{1/4} \sqrt{a + b x^2 + c x^4}\right)$$

Result (type 4, 473 leaves):

$$\frac{1}{6 \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x^3 \sqrt{a + b x^2 + c x^4}} \left(2 \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} (-a^2 - 5 a b x^2 - 4 b^2 x^4 - 3 b c x^6 + c^2 x^8) + \right.$$

$$4 i b (-b + \sqrt{b^2 - 4 a c}) x^3 \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \left.
$$\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right] -$$

$$i (-b^2 + 4 a c + 4 b \sqrt{b^2 - 4 a c}) x^3 \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \left.
$$\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right]$$$$$$

Problem 952: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b x^2 + c x^4)^{3/2}}{x^6} dx$$

Optimal (type 4, 400 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{(b^2 + 12ac) \sqrt{a + bx^2 + cx^4}}{5ax} + \frac{\sqrt{c} (b^2 + 12ac) x \sqrt{a + bx^2 + cx^4}}{5a(\sqrt{a} + \sqrt{c}x^2)} - \frac{(b - 6cx^2) \sqrt{a + bx^2 + cx^4}}{5x^3} \\
 & \frac{(a + bx^2 + cx^4)^{3/2}}{5x^5} - \left(c^{1/4} (b^2 + 12ac) (\sqrt{a} + \sqrt{c}x^2) \sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{c}x^2)^2}} \right. \\
 & \quad \left. \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c^{1/4}x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a}\sqrt{c}} \right) \right] \right) / \left(5a^{3/4} \sqrt{a + bx^2 + cx^4} \right) + \\
 & \left(c^{1/4} (b^2 + 8\sqrt{a}b\sqrt{c} + 12ac) (\sqrt{a} + \sqrt{c}x^2) \sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{c}x^2)^2}} \right. \\
 & \quad \left. \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4}x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a}\sqrt{c}} \right) \right] \right) / \left(10a^{3/4} \sqrt{a + bx^2 + cx^4} \right)
 \end{aligned}$$

Result (type 4, 527 leaves):

$$\begin{aligned}
 & \frac{1}{20a \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x^5 \sqrt{a + bx^2 + cx^4}} \\
 & \left(-4 \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} (a^3 + b^2x^6(b + cx^2) + a^2(3bx^2 + 8cx^4) + a(3b^2x^4 + 9bcx^6 + 7c^2x^8)) + \right. \\
 & \quad \left. i (b^2 + 12ac) (-b + \sqrt{b^2 - 4ac}) x^5 \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{\frac{2b - 2\sqrt{b^2 - 4ac} + 4cx^2}{b - \sqrt{b^2 - 4ac}}} \right. \\
 & \quad \left. \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] - \right. \\
 & \quad \left. i (-b^3 + 4abc + b^2\sqrt{b^2 - 4ac} + 12ac\sqrt{b^2 - 4ac}) x^5 \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{b + \sqrt{b^2 - 4ac}}} \right. \\
 & \quad \left. \sqrt{\frac{2b - 2\sqrt{b^2 - 4ac} + 4cx^2}{b - \sqrt{b^2 - 4ac}}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] \right)
 \end{aligned}$$

Problem 953: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + bx^2 + cx^4)^{3/2}}{x^8} dx$$

Optimal (type 4, 447 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{(b^2 - 20 a c) \sqrt{a + b x^2 + c x^4}}{35 a x^3} + \frac{2 b (b^2 - 8 a c) \sqrt{a + b x^2 + c x^4}}{35 a^2 x} \\
 & - \frac{2 b \sqrt{c} (b^2 - 8 a c) x \sqrt{a + b x^2 + c x^4}}{35 a^2 (\sqrt{a} + \sqrt{c} x^2)} - \frac{3 (b + 10 c x^2) \sqrt{a + b x^2 + c x^4}}{35 x^5} \\
 & - \frac{(a + b x^2 + c x^4)^{3/2}}{7 x^7} + \left(2 b c^{1/4} (b^2 - 8 a c) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right. \\
 & \left. \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(35 a^{7/4} \sqrt{a + b x^2 + c x^4} \right) - \\
 & \left(c^{1/4} (\sqrt{a} \sqrt{c} (b^2 - 20 a c) + 2 b (b^2 - 8 a c)) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right. \\
 & \left. \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(70 a^{7/4} \sqrt{a + b x^2 + c x^4} \right)
 \end{aligned}$$

Result (type 4, 572 leaves):

$$\begin{aligned}
 & \frac{1}{70 a^2 \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x^7 \sqrt{a + b x^2 + c x^4}} \\
 & \left(-2 \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} (5 a^4 - 2 b^3 x^8 (b + c x^2) + a^3 (13 b x^2 + 20 c x^4) + \right. \\
 & \quad \left. a b x^6 (-b^2 + 17 b c x^2 + 16 c^2 x^4) + 3 a^2 (3 b^2 x^4 + 13 b c x^6 + 5 c^2 x^8)) - \right. \\
 & \quad \left. i b (b^2 - 8 a c) (-b + \sqrt{b^2 - 4 a c}) x^7 \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \right. \\
 & \quad \left. \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. i (-b^4 + 9 a b^2 c - 20 a^2 c^2 + b^3 \sqrt{b^2 - 4 a c} - 8 a b c \sqrt{b^2 - 4 a c}) x^7 \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \right. \\
 & \quad \left. \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] \right)
 \end{aligned}$$

Problem 954: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{3 - 2x^2 - x^4} \, dx$$

Optimal (type 4, 48 leaves, 5 steps):

$$\frac{1}{3} x \sqrt{3 - 2x^2 - x^4} - \frac{2 \operatorname{EllipticE}[\operatorname{ArcSin}[x], -\frac{1}{3}]}{\sqrt{3}} + \frac{4 \operatorname{EllipticF}[\operatorname{ArcSin}[x], -\frac{1}{3}]}{\sqrt{3}}$$

Result (type 4, 59 leaves):

$$\frac{1}{3} \left(x \sqrt{3 - 2x^2 - x^4} - 2 i \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{x}{\sqrt{3}}\right], -3\right] - 4 i \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{x}{\sqrt{3}}\right], -3\right] \right)$$

Problem 963: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a + b x^2 + c x^4}} \, dx$$

Optimal (type 4, 313 leaves, 4 steps):

$$\frac{x \sqrt{a + b x^2 + c x^4}}{3 c} - \frac{2 b x \sqrt{a + b x^2 + c x^4}}{3 c^{3/2} (\sqrt{a} + \sqrt{c} x^2)} + \left(2 a^{1/4} b (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right) / \left(3 c^{7/4} \sqrt{a + b x^2 + c x^4} \right) - \left(a^{1/4} (2 b + \sqrt{a} \sqrt{c}) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right] \right) / \left(6 c^{7/4} \sqrt{a + b x^2 + c x^4} \right)$$

Result (type 4, 444 leaves):

$$\begin{aligned}
 & \left(2 c \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x (a + b x^2 + c x^4) - \right. \\
 & \quad i b \left(-b + \sqrt{b^2 - 4 a c} \right) \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \\
 & \quad \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] + \\
 & \quad i \left(-b^2 + a c + b \sqrt{b^2 - 4 a c} \right) \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \\
 & \quad \left. \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(6 c^2 \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \sqrt{a + b x^2 + c x^4} \right)
 \end{aligned}$$

Problem 964: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{a + b x^2 + c x^4}} dx$$

Optimal (type 4, 267 leaves, 3 steps):

$$\begin{aligned}
 & \frac{x \sqrt{a + b x^2 + c x^4}}{\sqrt{c} \left(\sqrt{a} + \sqrt{c} x^2 \right)} - \\
 & \left(a^{1/4} \left(\sqrt{a} + \sqrt{c} x^2 \right) \sqrt{\frac{a + b x^2 + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2 \right)^2}} \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \\
 & \left(c^{3/4} \sqrt{a + b x^2 + c x^4} \right) + \\
 & \left(a^{1/4} \left(\sqrt{a} + \sqrt{c} x^2 \right) \sqrt{\frac{a + b x^2 + c x^4}{\left(\sqrt{a} + \sqrt{c} x^2 \right)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \\
 & \left(2 c^{3/4} \sqrt{a + b x^2 + c x^4} \right)
 \end{aligned}$$

Result (type 4, 278 leaves):

$$\left(\left(-b + \sqrt{b^2 - 4ac} \right) \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \right. \\ \left. \left(\text{EllipticE} \left[\text{i ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] - \right. \right. \\ \left. \left. \text{EllipticF} \left[\text{i ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] \right) \right) / \\ \left(2\sqrt{2} c \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \sqrt{a + bx^2 + cx^4} \right)$$

Problem 965: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + bx^2 + cx^4}} dx$$

Optimal (type 4, 114 leaves, 1 step):

$$\left(\left(\sqrt{a} + \sqrt{c} x^2 \right) \sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \\ \left(2 a^{1/4} c^{1/4} \sqrt{a + bx^2 + cx^4} \right)$$

Result (type 4, 186 leaves):

$$- \left(\left(\text{i} \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \right. \right. \\ \left. \left. \text{EllipticF} \left[\text{i ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] \right) \right) / \\ \left(\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \sqrt{a + bx^2 + cx^4} \right)$$

Problem 966: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{a + bx^2 + cx^4}} dx$$

Optimal (type 4, 294 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+bx^2+cx^4}}{ax} + \frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a(\sqrt{a}+\sqrt{c}x^2)} - \\
 & \left(c^{1/4}(\sqrt{a}+\sqrt{c}x^2) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}} \operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \\
 & \left(a^{3/4}\sqrt{a+bx^2+cx^4} \right) + \\
 & \left(c^{1/4}(\sqrt{a}+\sqrt{c}x^2) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}} \operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \\
 & \left(2a^{3/4}\sqrt{a+bx^2+cx^4} \right)
 \end{aligned}$$

Result (type 4, 298 leaves):

$$\begin{aligned}
 & \left(-\frac{4(a+bx^2+cx^4)}{x} + \frac{1}{\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}} i\sqrt{2}(-b+\sqrt{b^2-4ac}) \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}} \right. \\
 & \left. \sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}} \left(\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right]x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} \right) - \right. \\
 & \left. \operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right]x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} \right] \right) / \left(4a\sqrt{a+bx^2+cx^4} \right)
 \end{aligned}$$

Problem 967: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}} dx$$

Optimal (type 4, 345 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x} - \frac{2b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a^2(\sqrt{a}+\sqrt{c}x^2)} + \\
 & \left(2bc^{1/4}(\sqrt{a}+\sqrt{c}x^2) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}} \text{EllipticE}\left[2\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \\
 & \left(3a^{7/4}\sqrt{a+bx^2+cx^4} \right) - \left((2b+\sqrt{a}\sqrt{c})c^{1/4}(\sqrt{a}+\sqrt{c}x^2) \sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}} \right. \\
 & \left. \text{EllipticF}\left[2\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right], \frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right] \right) / \left(6a^{7/4}\sqrt{a+bx^2+cx^4} \right)
 \end{aligned}$$

Result (type 4, 459 leaves):

$$\begin{aligned}
 & \frac{1}{6a^2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}} x^3 \sqrt{a+bx^2+cx^4} \left(-2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}} (a-2bx^2)(a+bx^2+cx^4) - \right. \\
 & \left. i b(-b+\sqrt{b^2-4ac}) x^3 \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}} \sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}} \right. \\
 & \left. \text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] + \right. \\
 & \left. i(-b^2+ac+b\sqrt{b^2-4ac}) x^3 \sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}} \sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}} \right. \\
 & \left. \text{EllipticF}\left[i\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right], \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right] \right)
 \end{aligned}$$

Problem 968: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}} dx$$

Optimal (type 3, 124 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{x^4\sqrt{a+bx^2-cx^4}}{6c} - \frac{(15b^2+16ac+10bcx^2)\sqrt{a+bx^2-cx^4}}{48c^3} - \\
 & \frac{b(5b^2+12ac)\text{ArcTan}\left[\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right]}{32c^{7/2}}
 \end{aligned}$$

Result (type 3, 112 leaves):

$$\frac{1}{96 c^{7/2}} \left(-2 \sqrt{c} \sqrt{a+b x^2-c x^4} (15 b^2+10 b c x^2+8 c (2 a+c x^4)) + 3 i (5 b^3+12 a b c) \operatorname{Log} \left[\frac{i (b-2 c x^2)}{\sqrt{c}} + 2 \sqrt{a+b x^2-c x^4} \right] \right)$$

Problem 969: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^5}{\sqrt{a+b x^2-c x^4}} dx$$

Optimal (type 3, 107 leaves, 5 steps):

$$-\frac{3 b \sqrt{a+b x^2-c x^4}}{8 c^2} - \frac{x^2 \sqrt{a+b x^2-c x^4}}{4 c} - \frac{(3 b^2+4 a c) \operatorname{ArcTan} \left[\frac{b-2 c x^2}{2 \sqrt{c} \sqrt{a+b x^2-c x^4}} \right]}{16 c^{5/2}}$$

Result (type 3, 94 leaves):

$$-\frac{(3 b+2 c x^2) \sqrt{a+b x^2-c x^4}}{8 c^2} + \frac{i (3 b^2+4 a c) \operatorname{Log} \left[\frac{i (b-2 c x^2)}{\sqrt{c}} + 2 \sqrt{a+b x^2-c x^4} \right]}{16 c^{5/2}}$$

Problem 970: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{a+b x^2-c x^4}} dx$$

Optimal (type 3, 70 leaves, 4 steps):

$$-\frac{\sqrt{a+b x^2-c x^4}}{2 c} - \frac{b \operatorname{ArcTan} \left[\frac{b-2 c x^2}{2 \sqrt{c} \sqrt{a+b x^2-c x^4}} \right]}{4 c^{3/2}}$$

Result (type 3, 77 leaves):

$$-\frac{\sqrt{a+b x^2-c x^4}}{2 c} + \frac{i b \operatorname{Log} \left[-\frac{i (-b+2 c x^2)}{\sqrt{c}} + 2 \sqrt{a+b x^2-c x^4} \right]}{4 c^{3/2}}$$

Problem 971: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{a+b x^2-c x^4}} dx$$

Optimal (type 3, 44 leaves, 3 steps):

$$-\frac{\operatorname{ArcTan} \left[\frac{b-2 c x^2}{2 \sqrt{c} \sqrt{a+b x^2-c x^4}} \right]}{2 \sqrt{c}}$$

Result (type 3, 51 leaves):

$$\frac{i \operatorname{Log}\left[-\frac{i(-b+2cx^2)}{\sqrt{c}} + 2\sqrt{a+bx^2-cx^4}\right]}{2\sqrt{c}}$$

Problem 976: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a+bx^2-cx^4}} dx$$

Optimal (type 4, 409 leaves, 5 steps):

$$\begin{aligned} & -\frac{x\sqrt{a+bx^2-cx^4}}{3c} - \left(b(b-\sqrt{b^2+4ac}) \sqrt{b+\sqrt{b^2+4ac}} \sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}} \sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}} \right. \\ & \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right], \frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right] \right) / \left(3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4} \right) + \\ & \left(\sqrt{b+\sqrt{b^2+4ac}} (b^2+ac-b\sqrt{b^2+4ac}) \sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}} \sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}} \right. \\ & \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right], \frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right] \right) / \left(3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4} \right) \end{aligned}$$

Result (type 4, 459 leaves):

$$\begin{aligned} & \frac{1}{6c^2 \sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}} \sqrt{a+bx^2-cx^4}} \\ & \left(2c \sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}} x(-a-bx^2+cx^4) - i\sqrt{2}b(-b+\sqrt{b^2+4ac}) \sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}} \right. \\ & \left. \sqrt{\frac{-b+\sqrt{b^2+4ac}+2cx^2}{-b+\sqrt{b^2+4ac}}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}} x\right], \frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right] \right) + \\ & i\sqrt{2}(-b^2-ac+b\sqrt{b^2+4ac}) \sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}} \sqrt{\frac{-b+\sqrt{b^2+4ac}+2cx^2}{-b+\sqrt{b^2+4ac}}} \\ & \left. \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}} x\right], \frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right] \right) \end{aligned}$$

Problem 977: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{a + b x^2 - c x^4}} dx$$

Optimal (type 4, 377 leaves, 4 steps):

$$\begin{aligned}
 & - \left(\left((b - \sqrt{b^2 + 4ac}) \sqrt{b + \sqrt{b^2 + 4ac}} \sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}} \sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}} \right. \right. \\
 & \quad \left. \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 + 4ac}}} \right], \frac{b + \sqrt{b^2 + 4ac}}{b - \sqrt{b^2 + 4ac}} \right] \right) / \left(2 \sqrt{2} c^{3/2} \sqrt{a + b x^2 - c x^4} \right) \right) + \\
 & \left((b - \sqrt{b^2 + 4ac}) \sqrt{b + \sqrt{b^2 + 4ac}} \sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}} \sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}} \right. \\
 & \quad \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2 + 4ac}}} \right], \frac{b + \sqrt{b^2 + 4ac}}{b - \sqrt{b^2 + 4ac}} \right] \right) / \left(2 \sqrt{2} c^{3/2} \sqrt{a + b x^2 - c x^4} \right)
 \end{aligned}$$

Result (type 4, 271 leaves):

$$\begin{aligned}
 & - \left(\left(i (-b + \sqrt{b^2 + 4ac}) \sqrt{1 + \frac{2cx^2}{-b + \sqrt{b^2 + 4ac}}} \sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}} \right. \right. \\
 & \quad \left(\text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{-\frac{c}{b + \sqrt{b^2 + 4ac}}} x \right], -\frac{b + \sqrt{b^2 + 4ac}}{-b + \sqrt{b^2 + 4ac}} \right] - \right. \\
 & \quad \left. \left. \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{-\frac{c}{b + \sqrt{b^2 + 4ac}}} x \right], -\frac{b + \sqrt{b^2 + 4ac}}{-b + \sqrt{b^2 + 4ac}} \right] \right) \right) / \\
 & \left(2 \sqrt{2} c \sqrt{-\frac{c}{b + \sqrt{b^2 + 4ac}}} \sqrt{a + b x^2 - c x^4} \right)
 \end{aligned}$$

Problem 978: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + b x^2 - c x^4}} dx$$

Optimal (type 4, 169 leaves, 2 steps):

$$\left(\sqrt{b + \sqrt{b^2 + 4ac}} \sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}} \sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 + 4ac}}}\right], \frac{b + \sqrt{b^2 + 4ac}}{b - \sqrt{b^2 + 4ac}}\right] \right) / \left(\sqrt{2}\sqrt{c}\sqrt{a + bx^2 - cx^4} \right)$$

Result (type 4, 177 leaves):

$$- \left(\left(i \sqrt{1 + \frac{2cx^2}{-b + \sqrt{b^2 + 4ac}}} \sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}} \right. \right. \\ \left. \left. \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2}\sqrt{-\frac{c}{b + \sqrt{b^2 + 4ac}}}x\right], -\frac{b + \sqrt{b^2 + 4ac}}{-b + \sqrt{b^2 + 4ac}}\right] \right) / \right. \\ \left. \left(\sqrt{2}\sqrt{-\frac{c}{b + \sqrt{b^2 + 4ac}}}\sqrt{a + bx^2 - cx^4} \right) \right)$$

Problem 979: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{a + bx^2 - cx^4}} dx$$

Optimal (type 4, 408 leaves, 6 steps):

$$- \frac{\sqrt{a + bx^2 - cx^4}}{ax} + \left((b - \sqrt{b^2 + 4ac}) \sqrt{b + \sqrt{b^2 + 4ac}} \sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}} \sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}} \right. \\ \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 + 4ac}}}\right], \frac{b + \sqrt{b^2 + 4ac}}{b - \sqrt{b^2 + 4ac}}\right] \right) / \left(2\sqrt{2}a\sqrt{c}\sqrt{a + bx^2 - cx^4} \right) - \\ \left((b - \sqrt{b^2 + 4ac}) \sqrt{b + \sqrt{b^2 + 4ac}} \sqrt{1 - \frac{2cx^2}{b - \sqrt{b^2 + 4ac}}} \sqrt{1 - \frac{2cx^2}{b + \sqrt{b^2 + 4ac}}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b + \sqrt{b^2 + 4ac}}}\right], \frac{b + \sqrt{b^2 + 4ac}}{b - \sqrt{b^2 + 4ac}}\right] \right) / \left(2\sqrt{2}a\sqrt{c}\sqrt{a + bx^2 - cx^4} \right)$$

Result (type 4, 283 leaves):

$$\left(-\frac{4 a}{x} - 4 b x + 4 c x^3 + \frac{1}{\sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}}} i \left(-b + \sqrt{b^2+4 a c} \right) \sqrt{2 + \frac{4 c x^2}{-b + \sqrt{b^2+4 a c}}} \right. \\ \left. \sqrt{1 - \frac{2 c x^2}{b + \sqrt{b^2+4 a c}}} \left(\text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{-\frac{c}{b + \sqrt{b^2+4 a c}}} x \right], \frac{b + \sqrt{b^2+4 a c}}{b - \sqrt{b^2+4 a c}} \right] - \right. \right. \\ \left. \left. \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{-\frac{c}{b + \sqrt{b^2+4 a c}}} x \right], \frac{b + \sqrt{b^2+4 a c}}{b - \sqrt{b^2+4 a c}} \right] \right) \right) / \left(4 a \sqrt{a + b x^2 - c x^4} \right)$$

Problem 980: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \sqrt{a + b x^2 - c x^4}} dx$$

Optimal (type 4, 445 leaves, 6 steps):

$$-\frac{\sqrt{a + b x^2 - c x^4}}{3 a x^3} + \frac{2 b \sqrt{a + b x^2 - c x^4}}{3 a^2 x} - \\ \left(b \left(b - \sqrt{b^2+4 a c} \right) \sqrt{b + \sqrt{b^2+4 a c}} \sqrt{1 - \frac{2 c x^2}{b - \sqrt{b^2+4 a c}}} \sqrt{1 - \frac{2 c x^2}{b + \sqrt{b^2+4 a c}}} \right. \\ \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2+4 a c}}} \right], \frac{b + \sqrt{b^2+4 a c}}{b - \sqrt{b^2+4 a c}} \right] \right) / \left(3 \sqrt{2} a^2 \sqrt{c} \sqrt{a + b x^2 - c x^4} \right) + \\ \left(\sqrt{b + \sqrt{b^2+4 a c}} \left(b^2 + a c - b \sqrt{b^2+4 a c} \right) \sqrt{1 - \frac{2 c x^2}{b - \sqrt{b^2+4 a c}}} \sqrt{1 - \frac{2 c x^2}{b + \sqrt{b^2+4 a c}}} \right. \\ \left. \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{2} \sqrt{c} x}{\sqrt{b + \sqrt{b^2+4 a c}}} \right], \frac{b + \sqrt{b^2+4 a c}}{b - \sqrt{b^2+4 a c}} \right] \right) / \left(3 \sqrt{2} a^2 \sqrt{c} \sqrt{a + b x^2 - c x^4} \right)$$

Result (type 4, 472 leaves):

$$\frac{1}{6 a^2 \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} x^3 \sqrt{a+b x^2-c x^4}} \left(-2 \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} (a-2 b x^2) (a+b x^2-c x^4) - \right.$$

$$\left. i \sqrt{2} b (-b+\sqrt{b^2+4 a c}) x^3 \sqrt{\frac{b+\sqrt{b^2+4 a c}-2 c x^2}{b+\sqrt{b^2+4 a c}}} \sqrt{\frac{-b+\sqrt{b^2+4 a c}+2 c x^2}{-b+\sqrt{b^2+4 a c}}} \right.$$

$$\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} x\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right] +$$

$$i \sqrt{2} (-b^2-a c+b \sqrt{b^2+4 a c}) x^3 \sqrt{\frac{b+\sqrt{b^2+4 a c}-2 c x^2}{b+\sqrt{b^2+4 a c}}} \sqrt{\frac{-b+\sqrt{b^2+4 a c}+2 c x^2}{-b+\sqrt{b^2+4 a c}}} \left.$$

$$\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{-\frac{c}{b+\sqrt{b^2+4 a c}}} x\right], \frac{b+\sqrt{b^2+4 a c}}{b-\sqrt{b^2+4 a c}}\right] \right)$$

Problem 989: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6}{(a+b x^2+c x^4)^{3/2}} dx$$

Optimal (type 4, 408 leaves, 5 steps):

$$\frac{x^3 (2 a+b x^2)}{(b^2-4 a c) \sqrt{a+b x^2+c x^4}} - \frac{b x \sqrt{a+b x^2+c x^4}}{c (b^2-4 a c)} +$$

$$\frac{2 (b^2-3 a c) x \sqrt{a+b x^2+c x^4}}{c^{3/2} (b^2-4 a c) (\sqrt{a}+\sqrt{c} x^2)} - \left(2 a^{1/4} (b^2-3 a c) (\sqrt{a}+\sqrt{c} x^2) \sqrt{\frac{a+b x^2+c x^4}{(\sqrt{a}+\sqrt{c} x^2)^2}} \right.$$

$$\left. \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(c^{7/4} (b^2-4 a c) \sqrt{a+b x^2+c x^4} \right) +$$

$$\left(a^{1/4} (2 b^2+\sqrt{a} b \sqrt{c}-6 a c) (\sqrt{a}+\sqrt{c} x^2) \sqrt{\frac{a+b x^2+c x^4}{(\sqrt{a}+\sqrt{c} x^2)^2}} \right.$$

$$\left. \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(2 c^{7/4} (b^2-4 a c) \sqrt{a+b x^2+c x^4} \right)$$

Result (type 4, 489 leaves):

$$\begin{aligned}
 & \frac{1}{2 c^2 (-b^2 + 4 a c) \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \sqrt{a + b x^2 + c x^4}} \left(2 c \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x (b^2 x^2 + a (b - 2 c x^2)) - \right. \\
 & \quad i (b^2 - 3 a c) (-b + \sqrt{b^2 - 4 a c}) \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \\
 & \quad \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] + \\
 & \quad i \left(-b^3 + 4 a b c + b^2 \sqrt{b^2 - 4 a c} - 3 a c \sqrt{b^2 - 4 a c} \right) \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \\
 & \quad \left. \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] \right)
 \end{aligned}$$

Problem 990: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{(a + b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 342 leaves, 4 steps):

$$\begin{aligned}
 & \frac{x (2 a + b x^2)}{(b^2 - 4 a c) \sqrt{a + b x^2 + c x^4}} - \frac{b x \sqrt{a + b x^2 + c x^4}}{\sqrt{c} (b^2 - 4 a c) (\sqrt{a} + \sqrt{c} x^2)} + \\
 & \left(a^{1/4} b (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \\
 & \left(c^{3/4} (b^2 - 4 a c) \sqrt{a + b x^2 + c x^4} \right) - \\
 & \left(a^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \\
 & \left(2 (b - 2 \sqrt{a} \sqrt{c}) c^{3/4} \sqrt{a + b x^2 + c x^4} \right)
 \end{aligned}$$

Result (type 4, 452 leaves):

$$\left(4 c \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x (2 a + b x^2) - \right.$$

$$i b \left(-b + \sqrt{b^2 - 4 a c} \right) \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \right.$$

$$\text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] + \right.$$

$$i \left(-b^2 + 4 a c + b \sqrt{b^2 - 4 a c} \right) \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \right.$$

$$\left. \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] \right) /$$

$$\left(4 c (b^2 - 4 a c) \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \sqrt{a + b x^2 + c x^4} \right)$$

Problem 991: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{(a + b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 4, 341 leaves, 4 steps):

$$- \frac{x (b + 2 c x^2)}{(b^2 - 4 a c) \sqrt{a + b x^2 + c x^4}} + \frac{2 \sqrt{c} x \sqrt{a + b x^2 + c x^4}}{(b^2 - 4 a c) (\sqrt{a} + \sqrt{c} x^2)} -$$

$$\left(2 a^{1/4} c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) /$$

$$\left((b^2 - 4 a c) \sqrt{a + b x^2 + c x^4} \right) +$$

$$\left((\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + b x^2 + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) /$$

$$\left(2 a^{1/4} (b - 2 \sqrt{a} \sqrt{c}) c^{1/4} \sqrt{a + b x^2 + c x^4} \right)$$

Result (type 4, 437 leaves):

$$\begin{aligned}
 & \left(-2 \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x (b + 2cx^2) + \right. \\
 & \quad i \left(-b + \sqrt{b^2 - 4ac} \right) \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{\frac{2b - 2\sqrt{b^2 - 4ac} + 4cx^2}{b - \sqrt{b^2 - 4ac}}} \\
 & \quad \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] - \\
 & \quad i \sqrt{2} \sqrt{b^2 - 4ac} \sqrt{\frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{b + \sqrt{b^2 - 4ac}}} \\
 & \quad \left. \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left(2 (b^2 - 4ac) \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \sqrt{a + bx^2 + cx^4} \right)
 \end{aligned}$$

Problem 992: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + bx^2 + cx^4)^{3/2}} dx$$

Optimal (type 4, 353 leaves, 4 steps):

$$\begin{aligned}
 & \frac{x (b^2 - 2ac + bcx^2)}{a (b^2 - 4ac) \sqrt{a + bx^2 + cx^4}} - \frac{b \sqrt{c} x \sqrt{a + bx^2 + cx^4}}{a (b^2 - 4ac) (\sqrt{a} + \sqrt{c} x^2)} + \\
 & \left(b c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \\
 & \left(a^{3/4} (b^2 - 4ac) \sqrt{a + bx^2 + cx^4} \right) - \\
 & \left(c^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \\
 & \left(2 a^{3/4} (b - 2 \sqrt{a} \sqrt{c}) \sqrt{a + bx^2 + cx^4} \right)
 \end{aligned}$$

Result (type 4, 456 leaves):

$$\begin{aligned}
 & - \left(\left(-4 \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x (b^2 - 2ac + bcx^2) + \right. \right. \\
 & \quad \left. \left. i b (-b + \sqrt{b^2 - 4ac}) \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{\frac{2b - 2\sqrt{b^2 - 4ac} + 4cx^2}{b - \sqrt{b^2 - 4ac}}} \right. \right. \\
 & \quad \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] - \\
 & \quad \left. \left. i (-b^2 + 4ac + b\sqrt{b^2 - 4ac}) \sqrt{\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{b + \sqrt{b^2 - 4ac}}} \sqrt{\frac{2b - 2\sqrt{b^2 - 4ac} + 4cx^2}{b - \sqrt{b^2 - 4ac}}} \right. \right. \\
 & \quad \left. \left. \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} x \right], \frac{b + \sqrt{b^2 - 4ac}}{b - \sqrt{b^2 - 4ac}} \right] \right) / \right. \\
 & \quad \left. \left(4a (b^2 - 4ac) \sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}} \sqrt{a + bx^2 + cx^4} \right) \right)
 \end{aligned}$$

Problem 993: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 (a + bx^2 + cx^4)^{3/2}} dx$$

Optimal (type 4, 428 leaves, 5 steps):

$$\begin{aligned}
 & \frac{b^2 - 2ac + bcx^2}{a (b^2 - 4ac) x \sqrt{a + bx^2 + cx^4}} - \frac{2 (b^2 - 3ac) \sqrt{a + bx^2 + cx^4}}{a^2 (b^2 - 4ac) x} + \\
 & \frac{2 \sqrt{c} (b^2 - 3ac) x \sqrt{a + bx^2 + cx^4}}{a^2 (b^2 - 4ac) (\sqrt{a} + \sqrt{c} x^2)} - \left(2 c^{1/4} (b^2 - 3ac) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right. \\
 & \quad \left. \text{EllipticE} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(a^{7/4} (b^2 - 4ac) \sqrt{a + bx^2 + cx^4} \right) + \\
 & \left(c^{1/4} (2b^2 + \sqrt{a} b \sqrt{c} - 6ac) (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + bx^2 + cx^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \right. \\
 & \quad \left. \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}} \right) \right] \right) / \left(2 a^{7/4} (b^2 - 4ac) \sqrt{a + bx^2 + cx^4} \right)
 \end{aligned}$$

Result (type 4, 515 leaves):

$$\begin{aligned}
 & \frac{1}{2 a^2 (b^2 - 4 a c) \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}} x \sqrt{a + b x^2 + c x^4}}} \\
 & \left(2 \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} (-4 a^2 c + 2 b^2 x^2 (b + c x^2) + a (b^2 - 7 b c x^2 - 6 c^2 x^4)) - \right. \\
 & \quad \left. i (b^2 - 3 a c) (-b + \sqrt{b^2 - 4 a c}) x \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \right. \\
 & \quad \left. \text{EllipticE} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. i (-b^3 + 4 a b c + b^2 \sqrt{b^2 - 4 a c} - 3 a c \sqrt{b^2 - 4 a c}) x \right. \\
 & \quad \left. \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{\frac{2 b - 2 \sqrt{b^2 - 4 a c} + 4 c x^2}{b - \sqrt{b^2 - 4 a c}}} \right. \\
 & \quad \left. \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} x \right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}} \right] \right)
 \end{aligned}$$

Problem 1003: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4}{\sqrt{a + (2 + 2 b - 2 (1 + b)) x^2 + c x^4}} dx$$

Optimal (type 4, 108 leaves, 3 steps):

$$\frac{x \sqrt{a + c x^4}}{3 c} - \frac{a^{3/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a + c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF} \left[2 \text{ArcTan} \left[\frac{c^{1/4} x}{a^{1/4}} \right], \frac{1}{2} \right]}{6 c^{5/4} \sqrt{a + c x^4}}$$

Result (type 4, 92 leaves):

$$\frac{x (a + c x^4) + \frac{i a \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF} \left[i \text{ArcSinh} \left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x \right], -1 \right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}}{3 c \sqrt{a + c x^4}}$$

Problem 1005: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2}{\sqrt{a + (2 + 2 b - 2 (1 + b)) x^2 + c x^4}} dx$$

Optimal (type 4, 210 leaves, 4 steps):

$$\frac{x \sqrt{a+c x^4}}{\sqrt{c} (\sqrt{a} + \sqrt{c} x^2)} - \frac{a^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{c^{3/4} \sqrt{a+c x^4}} +$$

$$\frac{a^{1/4} (\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 c^{3/4} \sqrt{a+c x^4}}$$

Result (type 4, 104 leaves):

$$\left(i \sqrt{1 + \frac{c x^4}{a}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right) \right) /$$

$$\left(\left(\frac{i \sqrt{c}}{\sqrt{a}} \right)^{3/2} \sqrt{a+c x^4} \right)$$

Problem 1007: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + (2 + 2 b - 2 (1 + b)) x^2 + c x^4}} dx$$

Optimal (type 4, 88 leaves, 2 steps):

$$\frac{(\sqrt{a} + \sqrt{c} x^2) \sqrt{\frac{a+c x^4}{(\sqrt{a} + \sqrt{c} x^2)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{1/4} c^{1/4} \sqrt{a+c x^4}}$$

Result (type 4, 74 leaves):

$$\frac{i \sqrt{1 + \frac{c x^4}{a}} \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{a+c x^4}}$$

Problem 1009: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^2 \sqrt{a + (2 + 2 b - 2 (1 + b)) x^2 + c x^4}} dx$$

Optimal (type 4, 232 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{\sqrt{a+c x^4}}{a x} + \frac{\sqrt{c} x \sqrt{a+c x^4}}{a\left(\sqrt{a}+\sqrt{c} x^2\right)} - \frac{c^{1/4}\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{a^{3/4} \sqrt{a+c x^4}} + \\
 & \frac{c^{1/4}\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{2 a^{3/4} \sqrt{a+c x^4}}
 \end{aligned}$$

Result (type 4, 121 leaves):

$$\begin{aligned}
 & \frac{1}{\sqrt{a+c x^4}} \left(-\frac{a+c x^4}{a x} - i \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} \sqrt{1+\frac{c x^4}{a}} \right. \\
 & \left. \left(\operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] - \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right] \right) \right)
 \end{aligned}$$

Problem 1011: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^4 \sqrt{a+(2+2 b-2(1+b)) x^2+c x^4}} dx$$

Optimal (type 4, 110 leaves, 3 steps):

$$-\frac{\sqrt{a+c x^4}}{3 a x^3} - \frac{c^{3/4}\left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{6 a^{5/4} \sqrt{a+c x^4}}$$

Result (type 4, 95 leaves):

$$\begin{aligned}
 & -\frac{a+c x^4}{x^3} + \frac{i c \sqrt{1+\frac{c x^4}{a}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}} x\right], -1\right]}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}} \\
 & \frac{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}}{3 a \sqrt{a+c x^4}}
 \end{aligned}$$

Problem 1039: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{3-2 x^2-x^4}} dx$$

Optimal (type 4, 12 leaves, 2 steps):

$$\frac{\operatorname{EllipticF}\left[\operatorname{ArcSin}[x], -\frac{1}{3}\right]}{\sqrt{3}}$$

Result (type 4, 18 leaves):

$$-i \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{x}{\sqrt{3}}\right], -3\right]$$

Problem 1040: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-1+5x^2-x^4}} dx$$

Optimal (type 4, 39 leaves, 2 steps):

$$\frac{\operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\sqrt{\frac{2}{5+\sqrt{21}}}\right] x, \frac{1}{42} (21+5\sqrt{21})\right]}{21^{1/4}}$$

Result (type 4, 87 leaves):

$$\frac{1}{2\sqrt{-1+5x^2-x^4}} \sqrt{5-\sqrt{21}-2x^2} \sqrt{2+(-5+\sqrt{21})x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1}{2}(5+\sqrt{21})} x\right], \frac{23}{2}-\frac{5\sqrt{21}}{2}\right]$$

Problem 1062: Result is not expressed in closed-form.

$$\int \frac{x^{9/2}}{a+bx^2+cx^4} dx$$

Optimal (type 3, 389 leaves, 9 steps):

$$\frac{2x^{3/2}}{3c} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b-\sqrt{b^2-4ac})^{1/4}}\right]}{2^{3/4}c^{7/4}(-b-\sqrt{b^2-4ac})^{1/4}} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b+\sqrt{b^2-4ac})^{1/4}}\right]}{2^{3/4}c^{7/4}(-b+\sqrt{b^2-4ac})^{1/4}} +$$

$$\frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b-\sqrt{b^2-4ac})^{1/4}}\right]}{2^{3/4}c^{7/4}(-b-\sqrt{b^2-4ac})^{1/4}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b+\sqrt{b^2-4ac})^{1/4}}\right]}{2^{3/4}c^{7/4}(-b+\sqrt{b^2-4ac})^{1/4}}$$

Result (type 7, 80 leaves):

$$\frac{4x^{3/2} - 3 \operatorname{RootSum}\left[a + b \#1^4 + c \#1^8 \&, \frac{a \operatorname{Log}\left[\sqrt{x} - \#1\right] + b \operatorname{Log}\left[\sqrt{x} - \#1\right] \#1^4}{b \#1 + 2c \#1^5} \&\right]}{6c}$$

Problem 1063: Result is not expressed in closed-form.

$$\int \frac{x^{7/2}}{a+bx^2+cx^4} dx$$

Optimal (type 3, 385 leaves, 9 steps):

$$\frac{2\sqrt{x}}{c} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b-\sqrt{b^2-4ac})^{1/4}}\right]}{2^{1/4}c^{5/4}(-b-\sqrt{b^2-4ac})^{3/4}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b+\sqrt{b^2-4ac})^{1/4}}\right]}{2^{1/4}c^{5/4}(-b+\sqrt{b^2-4ac})^{3/4}} +$$

$$\frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b-\sqrt{b^2-4ac})^{1/4}}\right]}{2^{1/4}c^{5/4}(-b-\sqrt{b^2-4ac})^{3/4}} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b+\sqrt{b^2-4ac})^{1/4}}\right]}{2^{1/4}c^{5/4}(-b+\sqrt{b^2-4ac})^{3/4}}$$

Result (type 7, 80 leaves):

$$\frac{-4\sqrt{x} + \text{RootSum}\left[a + b\#1^4 + c\#1^8 \&, \frac{a \text{Log}\left[\sqrt{x} - \#1\right] + b \text{Log}\left[\sqrt{x} - \#1\right] \#1^4}{b\#1^3 + 2c\#1^7} \&\right]}{2c}$$

Problem 1064: Result is not expressed in closed-form.

$$\int \frac{x^{5/2}}{a + b x^2 + c x^4} dx$$

Optimal (type 3, 331 leaves, 8 steps):

$$\frac{(-b - \sqrt{b^2 - 4ac})^{3/4} \text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b - \sqrt{b^2 - 4ac})^{1/4}}\right]}{2^{3/4}c^{3/4}\sqrt{b^2 - 4ac}} + \frac{(-b + \sqrt{b^2 - 4ac})^{3/4} \text{ArcTan}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b + \sqrt{b^2 - 4ac})^{1/4}}\right]}{2^{3/4}c^{3/4}\sqrt{b^2 - 4ac}} +$$

$$\frac{(-b - \sqrt{b^2 - 4ac})^{3/4} \text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b - \sqrt{b^2 - 4ac})^{1/4}}\right]}{2^{3/4}c^{3/4}\sqrt{b^2 - 4ac}} - \frac{(-b + \sqrt{b^2 - 4ac})^{3/4} \text{ArcTanh}\left[\frac{2^{1/4}c^{1/4}\sqrt{x}}{(-b + \sqrt{b^2 - 4ac})^{1/4}}\right]}{2^{3/4}c^{3/4}\sqrt{b^2 - 4ac}}$$

Result (type 7, 48 leaves):

$$\frac{1}{2} \text{RootSum}\left[a + b\#1^4 + c\#1^8 \&, \frac{\text{Log}\left[\sqrt{x} - \#1\right] \#1^3}{b + 2c\#1^4} \&\right]$$

Problem 1065: Result is not expressed in closed-form.

$$\int \frac{x^{3/2}}{a + b x^2 + c x^4} dx$$

Optimal (type 3, 331 leaves, 8 steps):

$$\frac{(-b - \sqrt{b^2 - 4ac})^{1/4} \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4ac})^{1/4}}\right]}{2^{1/4} c^{1/4} \sqrt{b^2 - 4ac}} - \frac{(-b + \sqrt{b^2 - 4ac})^{1/4} \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4ac})^{1/4}}\right]}{2^{1/4} c^{1/4} \sqrt{b^2 - 4ac}} +$$

$$\frac{(-b - \sqrt{b^2 - 4ac})^{1/4} \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4ac})^{1/4}}\right]}{2^{1/4} c^{1/4} \sqrt{b^2 - 4ac}} - \frac{(-b + \sqrt{b^2 - 4ac})^{1/4} \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4ac})^{1/4}}\right]}{2^{1/4} c^{1/4} \sqrt{b^2 - 4ac}}$$

Result (type 7, 46 leaves):

$$\frac{1}{2} \operatorname{RootSum}\left[a + b \#1^4 + c \#1^8 \&, \frac{\operatorname{Log}\left[\sqrt{x} - \#1\right] \#1}{b + 2c \#1^4} \&\right]$$

Problem 1066: Result is not expressed in closed-form.

$$\int \frac{\sqrt{x}}{a + b x^2 + c x^4} dx$$

Optimal (type 3, 331 leaves, 8 steps):

$$-\frac{2^{1/4} c^{1/4} \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4ac})^{1/4}}\right]}{\sqrt{b^2 - 4ac} (-b - \sqrt{b^2 - 4ac})^{1/4}} + \frac{2^{1/4} c^{1/4} \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4ac})^{1/4}}\right]}{\sqrt{b^2 - 4ac} (-b + \sqrt{b^2 - 4ac})^{1/4}} +$$

$$\frac{2^{1/4} c^{1/4} \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4ac})^{1/4}}\right]}{\sqrt{b^2 - 4ac} (-b - \sqrt{b^2 - 4ac})^{1/4}} - \frac{2^{1/4} c^{1/4} \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4ac})^{1/4}}\right]}{\sqrt{b^2 - 4ac} (-b + \sqrt{b^2 - 4ac})^{1/4}}$$

Result (type 7, 47 leaves):

$$\frac{1}{2} \operatorname{RootSum}\left[a + b \#1^4 + c \#1^8 \&, \frac{\operatorname{Log}\left[\sqrt{x} - \#1\right]}{b \#1 + 2c \#1^5} \&\right]$$

Problem 1067: Result is not expressed in closed-form.

$$\int \frac{1}{\sqrt{x} (a + b x^2 + c x^4)} dx$$

Optimal (type 3, 331 leaves, 8 steps):

$$\frac{2^{3/4} c^{3/4} \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b-\sqrt{b^2-4ac})^{1/4}}\right]}{\sqrt{b^2-4ac} (-b-\sqrt{b^2-4ac})^{3/4}} - \frac{2^{3/4} c^{3/4} \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b+\sqrt{b^2-4ac})^{1/4}}\right]}{\sqrt{b^2-4ac} (-b+\sqrt{b^2-4ac})^{3/4}} +$$

$$\frac{2^{3/4} c^{3/4} \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b-\sqrt{b^2-4ac})^{1/4}}\right]}{\sqrt{b^2-4ac} (-b-\sqrt{b^2-4ac})^{3/4}} - \frac{2^{3/4} c^{3/4} \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b+\sqrt{b^2-4ac})^{1/4}}\right]}{\sqrt{b^2-4ac} (-b+\sqrt{b^2-4ac})^{3/4}}$$

Result (type 7, 49 leaves):

$$\frac{1}{2} \operatorname{RootSum}\left[a + b \#1^4 + c \#1^8 \&, \frac{\operatorname{Log}\left[\sqrt{x} - \#1\right]}{b \#1^3 + 2 c \#1^7} \&\right]$$

Problem 1068: Result is not expressed in closed-form.

$$\int \frac{1}{x^{3/2} (a + b x^2 + c x^4)} dx$$

Optimal (type 3, 371 leaves, 9 steps):

$$\frac{2}{a \sqrt{x}} - \frac{c^{1/4} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b-\sqrt{b^2-4ac})^{1/4}}\right]}{2^{3/4} a (-b-\sqrt{b^2-4ac})^{1/4}} - \frac{c^{1/4} \left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b+\sqrt{b^2-4ac})^{1/4}}\right]}{2^{3/4} a (-b+\sqrt{b^2-4ac})^{1/4}} +$$

$$\frac{c^{1/4} \left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b-\sqrt{b^2-4ac})^{1/4}}\right]}{2^{3/4} a (-b-\sqrt{b^2-4ac})^{1/4}} + \frac{c^{1/4} \left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b+\sqrt{b^2-4ac})^{1/4}}\right]}{2^{3/4} a (-b+\sqrt{b^2-4ac})^{1/4}}$$

Result (type 7, 78 leaves):

$$\frac{\frac{4}{\sqrt{x}} + \operatorname{RootSum}\left[a + b \#1^4 + c \#1^8 \&, \frac{b \operatorname{Log}\left[\sqrt{x} - \#1\right] + c \operatorname{Log}\left[\sqrt{x} - \#1\right] \#1^4}{b \#1 + 2 c \#1^5} \&\right]}{2 a}$$

Problem 1069: Result is not expressed in closed-form.

$$\int \frac{1}{x^{5/2} (a + b x^2 + c x^4)} dx$$

Optimal (type 3, 371 leaves, 9 steps):

$$\begin{aligned}
 & -\frac{2}{3 a x^{3/2}} + \frac{c^{3/4} \left(1 - \frac{b}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{2^{1/4} a (-b - \sqrt{b^2 - 4 a c})^{3/4}} + \frac{c^{3/4} \left(1 + \frac{b}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{2^{1/4} a (-b + \sqrt{b^2 - 4 a c})^{3/4}} + \\
 & \frac{c^{3/4} \left(1 - \frac{b}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{2^{1/4} a (-b - \sqrt{b^2 - 4 a c})^{3/4}} + \frac{c^{3/4} \left(1 + \frac{b}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{2^{1/4} a (-b + \sqrt{b^2 - 4 a c})^{3/4}}
 \end{aligned}$$

Result (type 7, 82 leaves):

$$\frac{\frac{4}{x^{3/2}} + 3 \operatorname{RootSum}\left[a + b \#1^4 + c \#1^8 \&, \frac{b \operatorname{Log}\left[\sqrt{x} - \#1\right] + c \operatorname{Log}\left[\sqrt{x} - \#1\right] \#1^4}{b \#1^3 + 2 c \#1^7} \&\right]}{6 a}$$

Problem 1070: Result is not expressed in closed-form.

$$\int \frac{1}{x^{7/2} (a + b x^2 + c x^4)} dx$$

Optimal (type 3, 412 leaves, 10 steps):

$$\begin{aligned}
 & -\frac{2}{5 a x^{5/2}} + \frac{2 b}{a^2 \sqrt{x}} + \frac{c^{1/4} \left(b - \frac{b^2 - 2 a c}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{2^{3/4} a^2 (-b - \sqrt{b^2 - 4 a c})^{1/4}} + \\
 & \frac{c^{1/4} \left(b + \frac{b^2 - 2 a c}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{2^{3/4} a^2 (-b + \sqrt{b^2 - 4 a c})^{1/4}} - \\
 & \frac{c^{1/4} \left(b - \frac{b^2 - 2 a c}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{2^{3/4} a^2 (-b - \sqrt{b^2 - 4 a c})^{1/4}} - \frac{c^{1/4} \left(b + \frac{b^2 - 2 a c}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{2^{3/4} a^2 (-b + \sqrt{b^2 - 4 a c})^{1/4}}
 \end{aligned}$$

Result (type 7, 107 leaves):

$$\begin{aligned}
 & -\frac{1}{10 a^2} \left(\frac{4 a}{x^{5/2}} - \frac{20 b}{\sqrt{x}} - \right. \\
 & \left. 5 \operatorname{RootSum}\left[a + b \#1^4 + c \#1^8 \&, \frac{b^2 \operatorname{Log}\left[\sqrt{x} - \#1\right] - a c \operatorname{Log}\left[\sqrt{x} - \#1\right] + b c \operatorname{Log}\left[\sqrt{x} - \#1\right] \#1^4}{b \#1 + 2 c \#1^5} \&\right] \right)
 \end{aligned}$$

Problem 1071: Result is not expressed in closed-form.

$$\int \frac{x^{13/2}}{(a+b x^2+c x^4)^2} dx$$

Optimal (type 3, 544 leaves, 10 steps):

$$\begin{aligned} & -\frac{b x^{3/2}}{2 c (b^2-4 a c)} + \frac{x^{7/2} (2 a+b x^2)}{2 (b^2-4 a c) (a+b x^2+c x^4)} + \\ & \frac{(3 b^3-20 a b c+(3 b^2-14 a c) \sqrt{b^2-4 a c}) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b-\sqrt{b^2-4 a c})^{1/4}}\right]}{4 \times 2^{3/4} c^{7/4} (b^2-4 a c)^{3/2} (-b-\sqrt{b^2-4 a c})^{1/4}} - \\ & \frac{(3 b^3-20 a b c-(3 b^2-14 a c) \sqrt{b^2-4 a c}) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b+\sqrt{b^2-4 a c})^{1/4}}\right]}{4 \times 2^{3/4} c^{7/4} (b^2-4 a c)^{3/2} (-b+\sqrt{b^2-4 a c})^{1/4}} - \\ & \frac{(3 b^3-20 a b c+(3 b^2-14 a c) \sqrt{b^2-4 a c}) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b-\sqrt{b^2-4 a c})^{1/4}}\right]}{4 \times 2^{3/4} c^{7/4} (b^2-4 a c)^{3/2} (-b-\sqrt{b^2-4 a c})^{1/4}} + \\ & \frac{(3 b^3-20 a b c-(3 b^2-14 a c) \sqrt{b^2-4 a c}) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b+\sqrt{b^2-4 a c})^{1/4}}\right]}{4 \times 2^{3/4} c^{7/4} (b^2-4 a c)^{3/2} (-b+\sqrt{b^2-4 a c})^{1/4}} \end{aligned}$$

Result (type 7, 144 leaves):

$$\begin{aligned} & \frac{1}{8 c (b^2-4 a c)} \left(-\frac{4 x^{3/2} (b^2 x^2+a (b-2 c x^2))}{a+b x^2+c x^4} + \operatorname{RootSum}\left[a+b \#1^4+c \#1^8 \& , \right. \right. \\ & \left. \left. \frac{1}{b \#1+2 c \#1^5} \left(3 a b \operatorname{Log}\left[\sqrt{x}-\#1\right]+3 b^2 \operatorname{Log}\left[\sqrt{x}-\#1\right] \#1^4-14 a c \operatorname{Log}\left[\sqrt{x}-\#1\right] \#1^4\right) \& \right] \right) \end{aligned}$$

Problem 1072: Result is not expressed in closed-form.

$$\int \frac{x^{11/2}}{(a+b x^2+c x^4)^2} dx$$

Optimal (type 3, 520 leaves, 10 steps):

$$\begin{aligned}
 & -\frac{b \sqrt{x}}{2 c\left(b^2-4 a c\right)}+\frac{x^{5 / 2}\left(2 a+b x^2\right)}{2\left(b^2-4 a c\right)\left(a+b x^2+c x^4\right)}-\frac{\left(b^2-10 a c+\frac{b\left(b^2-12 a c\right)}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{2^{1 / 4} c^{1 / 4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1 / 4}}\right]}{4 \times 2^{1 / 4} c^{5 / 4}\left(b^2-4 a c\right)\left(-b-\sqrt{b^2-4 a c}\right)^{3 / 4}} \\
 & \frac{\left(b^2-10 a c-\frac{b\left(b^2-12 a c\right)}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{2^{1 / 4} c^{1 / 4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1 / 4}}\right]}{4 \times 2^{1 / 4} c^{5 / 4}\left(b^2-4 a c\right)\left(-b+\sqrt{b^2-4 a c}\right)^{3 / 4}} \\
 & \frac{\left(b^2-10 a c+\frac{b\left(b^2-12 a c\right)}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTanh}\left[\frac{2^{1 / 4} c^{1 / 4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1 / 4}}\right]}{4 \times 2^{1 / 4} c^{5 / 4}\left(b^2-4 a c\right)\left(-b-\sqrt{b^2-4 a c}\right)^{3 / 4}} \\
 & \frac{\left(b^2-10 a c-\frac{b\left(b^2-12 a c\right)}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTanh}\left[\frac{2^{1 / 4} c^{1 / 4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1 / 4}}\right]}{4 \times 2^{1 / 4} c^{5 / 4}\left(b^2-4 a c\right)\left(-b+\sqrt{b^2-4 a c}\right)^{3 / 4}}
 \end{aligned}$$

Result (type 7, 144 leaves):

$$\frac{1}{8 c\left(b^2-4 a c\right)}\left(-\frac{4 \sqrt{x}\left(b^2 x^2+a\left(b-2 c x^2\right)\right)}{a+b x^2+c x^4}+\operatorname{RootSum}\left[a+b \# 1^4+c \# 1^8 \&, \frac{a b \operatorname{Log}\left[\sqrt{x}-\# 1\right]+b^2 \operatorname{Log}\left[\sqrt{x}-\# 1\right] \# 1^4-10 a c \operatorname{Log}\left[\sqrt{x}-\# 1\right] \# 1^4}{b \# 1^3+2 c \# 1^7} \&\right]\right)$$

Problem 1073: Result is not expressed in closed-form.

$$\int \frac{x^{9 / 2}}{\left(a+b x^2+c x^4\right)^2} d x$$

Optimal (type 3, 471 leaves, 9 steps):

$$\frac{x^{3/2} (2 a + b x^2)}{2 (b^2 - 4 a c) (a + b x^2 + c x^4)} + \frac{(b^2 + 12 a c + b \sqrt{b^2 - 4 a c}) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{4 \times 2^{3/4} c^{3/4} (b^2 - 4 a c)^{3/2} (-b - \sqrt{b^2 - 4 a c})^{1/4}} +$$

$$\frac{\left(b - \frac{b^2 + 12 a c}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{4 \times 2^{3/4} c^{3/4} (b^2 - 4 a c) (-b + \sqrt{b^2 - 4 a c})^{1/4}} -$$

$$\frac{(b^2 + 12 a c + b \sqrt{b^2 - 4 a c}) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{4 \times 2^{3/4} c^{3/4} (b^2 - 4 a c)^{3/2} (-b - \sqrt{b^2 - 4 a c})^{1/4}} - \frac{\left(b - \frac{b^2 + 12 a c}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{4 \times 2^{3/4} c^{3/4} (b^2 - 4 a c) (-b + \sqrt{b^2 - 4 a c})^{1/4}}$$

Result (type 7, 124 leaves):

$$-\frac{-2 a x^{3/2} - b x^{7/2}}{2 (b^2 - 4 a c) (a + b x^2 + c x^4)} + \frac{\operatorname{RootSum}\left[a + b \#1^4 + c \#1^8 \&, \frac{-6 a \operatorname{Log}\left[\sqrt{x} - \#1\right] + b \operatorname{Log}\left[\sqrt{x} - \#1\right] \#1^4}{b \#1 + 2 c \#1^5} \&\right]}{8 (b^2 - 4 a c)}$$

Problem 1074: Result is not expressed in closed-form.

$$\int \frac{x^{7/2}}{(a + b x^2 + c x^4)^2} dx$$

Optimal (type 3, 483 leaves, 9 steps):

$$\frac{\sqrt{x} (2 a + b x^2)}{2 (b^2 - 4 a c) (a + b x^2 + c x^4)} - \frac{(3 b^2 + 4 a c + 3 b \sqrt{b^2 - 4 a c}) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{4 \times 2^{1/4} c^{1/4} (b^2 - 4 a c)^{3/2} (-b - \sqrt{b^2 - 4 a c})^{3/4}} +$$

$$\frac{(3 b^2 + 4 a c - 3 b \sqrt{b^2 - 4 a c}) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{4 \times 2^{1/4} c^{1/4} (b^2 - 4 a c)^{3/2} (-b + \sqrt{b^2 - 4 a c})^{3/4}} -$$

$$\frac{(3 b^2 + 4 a c + 3 b \sqrt{b^2 - 4 a c}) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{4 \times 2^{1/4} c^{1/4} (b^2 - 4 a c)^{3/2} (-b - \sqrt{b^2 - 4 a c})^{3/4}} +$$

$$\frac{(3 b^2 + 4 a c - 3 b \sqrt{b^2 - 4 a c}) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{4 \times 2^{1/4} c^{1/4} (b^2 - 4 a c)^{3/2} (-b + \sqrt{b^2 - 4 a c})^{3/4}}$$

Result (type 7, 127 leaves):

$$-\frac{-2 a \sqrt{x}-b x^{5 / 2}}{2\left(b^2-4 a c\right)\left(a+b x^2+c x^4\right)}+\frac{\text{RootSum}\left[a+b \sqrt[4]{c}+c \sqrt[8]{1}, \frac{-2 a \text{Log}\left[\sqrt{x}-\sqrt[4]{c}\right]+3 b \text{Log}\left[\sqrt{x}-\sqrt[4]{c}\right] \sqrt[4]{c}}{b \sqrt[4]{c}+2 c \sqrt[8]{1}}\right]}{8\left(b^2-4 a c\right)}$$

Problem 1075: Result is not expressed in closed-form.

$$\int \frac{x^{5 / 2}}{\left(a+b x^2+c x^4\right)^2} d x$$

Optimal (type 3, 450 leaves, 9 steps):

$$\begin{aligned} &-\frac{x^{3 / 2}\left(b+2 c x^2\right)}{2\left(b^2-4 a c\right)\left(a+b x^2+c x^4\right)}-\frac{c^{1 / 4}\left(4 b+\sqrt{b^2-4 a c}\right) \operatorname{ArcTan}\left[\frac{2^{1 / 4} c^{1 / 4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1 / 4}}\right]}{2 \times 2^{3 / 4}\left(b^2-4 a c\right)^{3 / 2}\left(-b-\sqrt{b^2-4 a c}\right)^{1 / 4}}+ \\ &\frac{c^{1 / 4}\left(4 b-\sqrt{b^2-4 a c}\right) \operatorname{ArcTan}\left[\frac{2^{1 / 4} c^{1 / 4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1 / 4}}\right]}{2 \times 2^{3 / 4}\left(b^2-4 a c\right)^{3 / 2}\left(-b+\sqrt{b^2-4 a c}\right)^{1 / 4}}+ \\ &\frac{c^{1 / 4}\left(4 b+\sqrt{b^2-4 a c}\right) \operatorname{ArcTanh}\left[\frac{2^{1 / 4} c^{1 / 4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1 / 4}}\right]}{2 \times 2^{3 / 4}\left(b^2-4 a c\right)^{3 / 2}\left(-b-\sqrt{b^2-4 a c}\right)^{1 / 4}}-\frac{c^{1 / 4}\left(4 b-\sqrt{b^2-4 a c}\right) \operatorname{ArcTanh}\left[\frac{2^{1 / 4} c^{1 / 4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1 / 4}}\right]}{2 \times 2^{3 / 4}\left(b^2-4 a c\right)^{3 / 2}\left(-b+\sqrt{b^2-4 a c}\right)^{1 / 4}} \end{aligned}$$

Result (type 7, 109 leaves):

$$\begin{aligned} &-\frac{1}{8\left(b^2-4 a c\right)} \\ &\left(\frac{4 x^{3 / 2}\left(b+2 c x^2\right)}{a+b x^2+c x^4}+\text{RootSum}\left[a+b \sqrt[4]{c}+c \sqrt[8]{1}, \frac{-3 b \text{Log}\left[\sqrt{x}-\sqrt[4]{c}\right]+2 c \text{Log}\left[\sqrt{x}-\sqrt[4]{c}\right] \sqrt[4]{c}}{b \sqrt[4]{c}+2 c \sqrt[8]{1}}\right]\right) \end{aligned}$$

Problem 1076: Result is not expressed in closed-form.

$$\int \frac{x^{3 / 2}}{\left(a+b x^2+c x^4\right)^2} d x$$

Optimal (type 3, 442 leaves, 9 steps):

$$\begin{aligned}
 & -\frac{\sqrt{x} (b+2 c x^2)}{2 (b^2-4 a c) (a+b x^2+c x^4)} + \frac{c^{3/4} \left(3+\frac{4 b}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b-\sqrt{b^2-4 a c})^{1/4}}\right]}{2 \times 2^{1/4} (b^2-4 a c) (-b-\sqrt{b^2-4 a c})^{3/4}} + \\
 & \frac{c^{3/4} \left(3-\frac{4 b}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b+\sqrt{b^2-4 a c})^{1/4}}\right]}{2 \times 2^{1/4} (b^2-4 a c) (-b+\sqrt{b^2-4 a c})^{3/4}} + \\
 & \frac{c^{3/4} \left(3+\frac{4 b}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b-\sqrt{b^2-4 a c})^{1/4}}\right]}{2 \times 2^{1/4} (b^2-4 a c) (-b-\sqrt{b^2-4 a c})^{3/4}} + \frac{c^{3/4} \left(3-\frac{4 b}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b+\sqrt{b^2-4 a c})^{1/4}}\right]}{2 \times 2^{1/4} (b^2-4 a c) (-b+\sqrt{b^2-4 a c})^{3/4}}
 \end{aligned}$$

Result (type 7, 111 leaves):

$$\begin{aligned}
 & -\frac{1}{8 (b^2-4 a c)} \\
 & \left(\frac{4 \sqrt{x} (b+2 c x^2)}{a+b x^2+c x^4} + \operatorname{RootSum}\left[a+b \#1^4+c \#1^8 \&, \frac{-b \operatorname{Log}\left[\sqrt{x}-\#1\right]+6 c \operatorname{Log}\left[\sqrt{x}-\#1\right] \#1^4}{b \#1^3+2 c \#1^7} \&\right] \right)
 \end{aligned}$$

Problem 1077: Result is not expressed in closed-form.

$$\int \frac{\sqrt{x}}{(a+b x^2+c x^4)^2} dx$$

Optimal (type 3, 489 leaves, 9 steps):

$$\begin{aligned}
 & \frac{x^{3/2} (b^2-2 a c+b c x^2)}{2 a (b^2-4 a c) (a+b x^2+c x^4)} + \frac{c^{1/4} \left(b-\frac{b^2-20 a c}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b-\sqrt{b^2-4 a c})^{1/4}}\right]}{4 \times 2^{3/4} a (b^2-4 a c) (-b-\sqrt{b^2-4 a c})^{1/4}} + \\
 & \frac{c^{1/4} \left(b+\frac{b^2-20 a c}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b+\sqrt{b^2-4 a c})^{1/4}}\right]}{4 \times 2^{3/4} a (b^2-4 a c) (-b+\sqrt{b^2-4 a c})^{1/4}} - \\
 & \frac{c^{1/4} \left(b-\frac{b^2-20 a c}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b-\sqrt{b^2-4 a c})^{1/4}}\right]}{4 \times 2^{3/4} a (b^2-4 a c) (-b-\sqrt{b^2-4 a c})^{1/4}} - \frac{c^{1/4} \left(b+\frac{b^2-20 a c}{\sqrt{b^2-4 a c}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b+\sqrt{b^2-4 a c})^{1/4}}\right]}{4 \times 2^{3/4} a (b^2-4 a c) (-b+\sqrt{b^2-4 a c})^{1/4}}
 \end{aligned}$$

Result (type 7, 149 leaves):

$$-\left(\left(4 x^{3/2} \left(b^2-2 a c+b c x^2\right)+\left(a+b x^2+c x^4\right) \operatorname{RootSum}\left[a+b \#1^4+c \#1^8 \&, \frac{1}{b \#1+2 c \#1^5}\left(b^2 \operatorname{Log}\left[\sqrt{x}-\#1\right]-10 a c \operatorname{Log}\left[\sqrt{x}-\#1\right]+b c \operatorname{Log}\left[\sqrt{x}-\#1\right] \#1^4\right) \&\right]\right) / \left(8 a\left(-b^2+4 a c\right)\left(a+b x^2+c x^4\right)\right)\right)$$

Problem 1078: Result is not expressed in closed-form.

$$\int \frac{1}{\sqrt{x} \left(a+b x^2+c x^4\right)^2} dx$$

Optimal (type 3, 503 leaves, 9 steps):

$$\frac{\sqrt{x} \left(b^2-2 a c+b c x^2\right)}{2 a\left(b^2-4 a c\right)\left(a+b x^2+c x^4\right)} + \frac{c^{3/4} \left(3 b^2-28 a c-3 b \sqrt{b^2-4 a c}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4 \times 2^{1/4} a\left(b^2-4 a c\right)^{3/2}\left(-b-\sqrt{b^2-4 a c}\right)^{3/4}} -$$

$$\frac{c^{3/4} \left(3 b^2-28 a c+3 b \sqrt{b^2-4 a c}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4 \times 2^{1/4} a\left(b^2-4 a c\right)^{3/2}\left(-b+\sqrt{b^2-4 a c}\right)^{3/4}} +$$

$$\frac{c^{3/4} \left(3 b^2-28 a c-3 b \sqrt{b^2-4 a c}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4 \times 2^{1/4} a\left(b^2-4 a c\right)^{3/2}\left(-b-\sqrt{b^2-4 a c}\right)^{3/4}} -$$

$$\frac{c^{3/4} \left(3 b^2-28 a c+3 b \sqrt{b^2-4 a c}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1/4}}\right]}{4 \times 2^{1/4} a\left(b^2-4 a c\right)^{3/2}\left(-b+\sqrt{b^2-4 a c}\right)^{3/4}}$$

Result (type 7, 153 leaves):

$$-\left(\left(4 \sqrt{x} \left(b^2-2 a c+b c x^2\right)+\left(a+b x^2+c x^4\right) \operatorname{RootSum}\left[a+b \#1^4+c \#1^8 \&, \frac{1}{b \#1^3+2 c \#1^7}\left(3 b^2 \operatorname{Log}\left[\sqrt{x}-\#1\right]-14 a c \operatorname{Log}\left[\sqrt{x}-\#1\right]+3 b c \operatorname{Log}\left[\sqrt{x}-\#1\right] \#1^4\right) \&\right]\right) / \left(8 a\left(-b^2+4 a c\right)\left(a+b x^2+c x^4\right)\right)\right)$$

Problem 1079: Result is not expressed in closed-form.

$$\int \frac{1}{x^{3/2} \left(a+b x^2+c x^4\right)^2} dx$$

Optimal (type 3, 573 leaves, 10 steps):

$$\begin{aligned}
 & -\frac{5 b^2-18 a c}{2 a^2\left(b^2-4 a c\right) \sqrt{x}}+\frac{b^2-2 a c+b c x^2}{2 a\left(b^2-4 a c\right) \sqrt{x}\left(a+b x^2+c x^4\right)}+ \\
 & \left(c^{1 / 4}\left(5 b^3-28 a b c-\left(5 b^2-18 a c\right) \sqrt{b^2-4 a c}\right) \operatorname{ArcTan}\left[\frac{2^{1 / 4} c^{1 / 4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1 / 4}}\right]\right) / \\
 & \left(4 \times 2^{3 / 4} a^2\left(b^2-4 a c\right)^{3 / 2}\left(-b-\sqrt{b^2-4 a c}\right)^{1 / 4}\right)- \\
 & \left(c^{1 / 4}\left(5 b^3-28 a b c+\left(5 b^2-18 a c\right) \sqrt{b^2-4 a c}\right) \operatorname{ArcTan}\left[\frac{2^{1 / 4} c^{1 / 4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1 / 4}}\right]\right) / \\
 & \left(4 \times 2^{3 / 4} a^2\left(b^2-4 a c\right)^{3 / 2}\left(-b+\sqrt{b^2-4 a c}\right)^{1 / 4}\right)- \\
 & \left(c^{1 / 4}\left(5 b^3-28 a b c-\left(5 b^2-18 a c\right) \sqrt{b^2-4 a c}\right) \operatorname{ArcTanh}\left[\frac{2^{1 / 4} c^{1 / 4} \sqrt{x}}{\left(-b-\sqrt{b^2-4 a c}\right)^{1 / 4}}\right]\right) / \\
 & \left(4 \times 2^{3 / 4} a^2\left(b^2-4 a c\right)^{3 / 2}\left(-b-\sqrt{b^2-4 a c}\right)^{1 / 4}\right)+ \\
 & \left(c^{1 / 4}\left(5 b^3-28 a b c+\left(5 b^2-18 a c\right) \sqrt{b^2-4 a c}\right) \operatorname{ArcTanh}\left[\frac{2^{1 / 4} c^{1 / 4} \sqrt{x}}{\left(-b+\sqrt{b^2-4 a c}\right)^{1 / 4}}\right]\right) / \\
 & \left(4 \times 2^{3 / 4} a^2\left(b^2-4 a c\right)^{3 / 2}\left(-b+\sqrt{b^2-4 a c}\right)^{1 / 4}\right)
 \end{aligned}$$

Result (type 7, 190 leaves):

$$\begin{aligned}
 & -\frac{1}{8 a^2}\left(\frac{16}{\sqrt{x}}+\frac{4 x^{3 / 2}\left(b^3-3 a b c+b^2 c x^2-2 a c^2 x^2\right)}{\left(b^2-4 a c\right)\left(a+b x^2+c x^4\right)}+\right. \\
 & \left.\frac{1}{b^2-4 a c} \operatorname{RootSum}\left[a+b \# 1^4+c \# 1^8 \&, \frac{1}{b \# 1+2 c \# 1^5}\left(5 b^3 \operatorname{Log}\left[\sqrt{x}-\# 1\right]-\right.\right.\right. \\
 & \left.\left.\left.23 a b c \operatorname{Log}\left[\sqrt{x}-\# 1\right]+5 b^2 c \operatorname{Log}\left[\sqrt{x}-\# 1\right] \# 1^4-18 a c^2 \operatorname{Log}\left[\sqrt{x}-\# 1\right] \# 1^4\right) \&\right]\right)
 \end{aligned}$$

Problem 1080: Result is not expressed in closed-form.

$$\int \frac{x^{15 / 2}}{\left(a+b x^2+c x^4\right)^3} d x$$

Optimal (type 3, 621 leaves, 11 steps):

$$\begin{aligned}
 & - \frac{3 (b^2 + 12 a c) \sqrt{x}}{16 c (b^2 - 4 a c)^2} + \frac{x^{9/2} (2 a + b x^2)}{4 (b^2 - 4 a c) (a + b x^2 + c x^4)^2} + \\
 & \frac{3 x^{5/2} (8 a b + (b^2 + 12 a c) x^2)}{16 (b^2 - 4 a c)^2 (a + b x^2 + c x^4)} - \frac{3 \left(b^3 - 28 a b c + \frac{b^4 - 30 a b^2 c - 24 a^2 c^2}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTan} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}} \right]}{32 \times 2^{1/4} c^{5/4} (b^2 - 4 a c)^2 (-b - \sqrt{b^2 - 4 a c})^{3/4}} - \\
 & \frac{3 \left(b^3 - 28 a b c - \frac{b^4 - 30 a b^2 c - 24 a^2 c^2}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTan} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}} \right]}{32 \times 2^{1/4} c^{5/4} (b^2 - 4 a c)^2 (-b + \sqrt{b^2 - 4 a c})^{3/4}} - \\
 & \frac{3 \left(b^3 - 28 a b c + \frac{b^4 - 30 a b^2 c - 24 a^2 c^2}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTanh} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}} \right]}{32 \times 2^{1/4} c^{5/4} (b^2 - 4 a c)^2 (-b - \sqrt{b^2 - 4 a c})^{3/4}} - \\
 & \frac{3 \left(b^3 - 28 a b c - \frac{b^4 - 30 a b^2 c - 24 a^2 c^2}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTanh} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}} \right]}{32 \times 2^{1/4} c^{5/4} (b^2 - 4 a c)^2 (-b + \sqrt{b^2 - 4 a c})^{3/4}}
 \end{aligned}$$

Result (type 7, 254 leaves):

$$\begin{aligned}
 & \left(4 \sqrt{x} (-4 b^4 + 21 a b^2 c - 68 a^2 c^2 + b^3 c x^2 - 28 a b c^2 x^2) (a + b x^2 + c x^4) + \right. \\
 & \left. 16 (b^2 - 4 a c) \sqrt{x} (-2 a^2 c + b^3 x^2 + a b (b - 3 c x^2)) + 3 c (a + b x^2 + c x^4)^2 \right. \\
 & \left. \text{RootSum} [a + b \#1^4 + c \#1^8 \&, \frac{1}{b \#1^3 + 2 c \#1^7} (a b^2 \text{Log} [\sqrt{x} - \#1] + 12 a^2 c \text{Log} [\sqrt{x} - \#1] + b^3 \right. \\
 & \left. \left. \text{Log} [\sqrt{x} - \#1] \#1^4 - 28 a b c \text{Log} [\sqrt{x} - \#1] \#1^4) \&] \right) / (64 c^2 (b^2 - 4 a c)^2 (a + b x^2 + c x^4)^2)
 \end{aligned}$$

Problem 1081: Result is not expressed in closed-form.

$$\int \frac{x^{13/2}}{(a + b x^2 + c x^4)^3} dx$$

Optimal (type 3, 569 leaves, 10 steps):

$$\begin{aligned}
 & \frac{x^{7/2} (2 a + b x^2)}{4 (b^2 - 4 a c) (a + b x^2 + c x^4)^2} + \frac{x^{3/2} (24 a b + (5 b^2 + 28 a c) x^2)}{16 (b^2 - 4 a c)^2 (a + b x^2 + c x^4)} + \\
 & \frac{\left(5 b^3 + 172 a b c + \sqrt{b^2 - 4 a c} (5 b^2 + 28 a c)\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{32 \times 2^{3/4} c^{3/4} (b^2 - 4 a c)^{5/2} (-b - \sqrt{b^2 - 4 a c})^{1/4}} + \\
 & \frac{\left(5 b^2 + 28 a c - \frac{5 b^3 + 172 a b c}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{32 \times 2^{3/4} c^{3/4} (b^2 - 4 a c)^2 (-b + \sqrt{b^2 - 4 a c})^{1/4}} - \\
 & \left(\left(5 b^3 + 172 a b c + \sqrt{b^2 - 4 a c} (5 b^2 + 28 a c)\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]\right) / \\
 & \left(32 \times 2^{3/4} c^{3/4} (b^2 - 4 a c)^{5/2} (-b - \sqrt{b^2 - 4 a c})^{1/4}\right) - \\
 & \frac{\left(5 b^2 + 28 a c - \frac{5 b^3 + 172 a b c}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{32 \times 2^{3/4} c^{3/4} (b^2 - 4 a c)^2 (-b + \sqrt{b^2 - 4 a c})^{1/4}}
 \end{aligned}$$

Result (type 7, 216 leaves):

$$\begin{aligned}
 & \left(4 x^{3/2} (4 b^3 + 8 a b c + 5 b^2 c x^2 + 28 a c^2 x^2) (a + b x^2 + c x^4) - \right. \\
 & \quad 16 (b^2 - 4 a c) x^{3/2} (b^2 x^2 + a (b - 2 c x^2)) + c (a + b x^2 + c x^4)^2 \operatorname{RootSum}\left[a + b \#1^4 + c \#1^8 \&, \right. \\
 & \quad \left. \frac{1}{b \#1 + 2 c \#1^5} \left(-72 a b \operatorname{Log}[\sqrt{x} - \#1] + 5 b^2 \operatorname{Log}[\sqrt{x} - \#1] \#1^4 + 28 a c \operatorname{Log}[\sqrt{x} - \#1] \#1^4\right) \&\right] \left. \right) / \\
 & (64 c (b^2 - 4 a c)^2 (a + b x^2 + c x^4)^2)
 \end{aligned}$$

Problem 1082: Result is not expressed in closed-form.

$$\int \frac{x^{11/2}}{(a + b x^2 + c x^4)^3} dx$$

Optimal (type 3, 569 leaves, 10 steps):

$$\frac{x^{5/2} (2 a + b x^2)}{4 (b^2 - 4 a c) (a + b x^2 + c x^4)^2} + \frac{\sqrt{x} (24 a b + (7 b^2 + 20 a c) x^2)}{16 (b^2 - 4 a c)^2 (a + b x^2 + c x^4)} -$$

$$\frac{3 (7 b^3 + 36 a b c + \sqrt{b^2 - 4 a c} (7 b^2 + 20 a c)) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{32 \times 2^{1/4} c^{1/4} (b^2 - 4 a c)^{5/2} (-b - \sqrt{b^2 - 4 a c})^{3/4}} -$$

$$\frac{3 \left(7 b^2 + 20 a c - \frac{7 b^3 + 36 a b c}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{32 \times 2^{1/4} c^{1/4} (b^2 - 4 a c)^2 (-b + \sqrt{b^2 - 4 a c})^{3/4}} -$$

$$\left(3 (7 b^3 + 36 a b c + \sqrt{b^2 - 4 a c} (7 b^2 + 20 a c)) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]\right) /$$

$$\left(32 \times 2^{1/4} c^{1/4} (b^2 - 4 a c)^{5/2} (-b - \sqrt{b^2 - 4 a c})^{3/4}\right) -$$

$$\frac{3 \left(7 b^2 + 20 a c - \frac{7 b^3 + 36 a b c}{\sqrt{b^2 - 4 a c}}\right) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{32 \times 2^{1/4} c^{1/4} (b^2 - 4 a c)^2 (-b + \sqrt{b^2 - 4 a c})^{3/4}}$$

Result (type 7, 219 leaves):

$$\left(4 \sqrt{x} (4 b^3 + 8 a b c + 7 b^2 c x^2 + 20 a c^2 x^2) (a + b x^2 + c x^4) -$$

$$16 (b^2 - 4 a c) \sqrt{x} (b^2 x^2 + a (b - 2 c x^2)) + 3 c (a + b x^2 + c x^4)^2 \operatorname{RootSum}\left[a + b \#1^4 + c \#1^8 \&, \right.$$

$$\left. \frac{1}{b \#1^3 + 2 c \#1^7} (-8 a b \operatorname{Log}[\sqrt{x} - \#1] + 7 b^2 \operatorname{Log}[\sqrt{x} - \#1] \#1^4 + 20 a c \operatorname{Log}[\sqrt{x} - \#1] \#1^4) \&\right] \Big) /$$

$$(64 c (b^2 - 4 a c)^2 (a + b x^2 + c x^4)^2)$$

Problem 1083: Result is not expressed in closed-form.

$$\int \frac{x^{9/2}}{(a + b x^2 + c x^4)^3} dx$$

Optimal (type 3, 533 leaves, 10 steps):

$$\begin{aligned}
 & \frac{x^{3/2} (2 a + b x^2)}{4 (b^2 - 4 a c) (a + b x^2 + c x^4)^2} - \frac{3 x^{3/2} (5 b^2 - 4 a c + 8 b c x^2)}{16 (b^2 - 4 a c)^2 (a + b x^2 + c x^4)} - \\
 & \frac{3 c^{1/4} (11 b^2 + 20 a c + 4 b \sqrt{b^2 - 4 a c}) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{16 \times 2^{3/4} (b^2 - 4 a c)^{5/2} (-b - \sqrt{b^2 - 4 a c})^{1/4}} + \\
 & \frac{3 c^{1/4} (11 b^2 + 20 a c - 4 b \sqrt{b^2 - 4 a c}) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{16 \times 2^{3/4} (b^2 - 4 a c)^{5/2} (-b + \sqrt{b^2 - 4 a c})^{1/4}} + \\
 & \frac{3 c^{1/4} (11 b^2 + 20 a c + 4 b \sqrt{b^2 - 4 a c}) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{16 \times 2^{3/4} (b^2 - 4 a c)^{5/2} (-b - \sqrt{b^2 - 4 a c})^{1/4}} - \\
 & \frac{3 c^{1/4} (11 b^2 + 20 a c - 4 b \sqrt{b^2 - 4 a c}) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{16 \times 2^{3/4} (b^2 - 4 a c)^{5/2} (-b + \sqrt{b^2 - 4 a c})^{1/4}}
 \end{aligned}$$

Result (type 7, 176 leaves):

$$\begin{aligned}
 & \frac{1}{64 (b^2 - 4 a c)^2} \\
 & \left(\frac{16 (b^2 - 4 a c) x^{3/2} (2 a + b x^2)}{(a + b x^2 + c x^4)^2} - \frac{12 x^{3/2} (5 b^2 - 4 a c + 8 b c x^2)}{a + b x^2 + c x^4} - 3 \operatorname{RootSum}\left[a + b \#1^4 + c \#1^8 \&, \right. \right. \\
 & \left. \left. \frac{1}{b \#1 + 2 c \#1^5} \left(-7 b^2 \operatorname{Log}\left[\sqrt{x} - \#1\right] - 20 a c \operatorname{Log}\left[\sqrt{x} - \#1\right] + 8 b c \operatorname{Log}\left[\sqrt{x} - \#1\right] \#1^4 \right) \& \right] \right)
 \end{aligned}$$

Problem 1084: Result is not expressed in closed-form.

$$\int \frac{x^{7/2}}{(a + b x^2 + c x^4)^3} dx$$

Optimal (type 3, 533 leaves, 10 steps):

$$\frac{\sqrt{x} (2 a + b x^2)}{4 (b^2 - 4 a c) (a + b x^2 + c x^4)^2} - \frac{\sqrt{x} (13 b^2 - 4 a c + 24 b c x^2)}{16 (b^2 - 4 a c)^2 (a + b x^2 + c x^4)} +$$

$$\frac{c^{3/4} (41 b^2 + 28 a c + 36 b \sqrt{b^2 - 4 a c}) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{16 \times 2^{1/4} (b^2 - 4 a c)^{5/2} (-b - \sqrt{b^2 - 4 a c})^{3/4}} -$$

$$\frac{c^{3/4} (41 b^2 + 28 a c - 36 b \sqrt{b^2 - 4 a c}) \operatorname{ArcTan}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{16 \times 2^{1/4} (b^2 - 4 a c)^{5/2} (-b + \sqrt{b^2 - 4 a c})^{3/4}} +$$

$$\frac{c^{3/4} (41 b^2 + 28 a c + 36 b \sqrt{b^2 - 4 a c}) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}}\right]}{16 \times 2^{1/4} (b^2 - 4 a c)^{5/2} (-b - \sqrt{b^2 - 4 a c})^{3/4}} -$$

$$\frac{c^{3/4} (41 b^2 + 28 a c - 36 b \sqrt{b^2 - 4 a c}) \operatorname{ArcTanh}\left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}}\right]}{16 \times 2^{1/4} (b^2 - 4 a c)^{5/2} (-b + \sqrt{b^2 - 4 a c})^{3/4}}$$

Result (type 7, 177 leaves):

$$-\frac{1}{64 (b^2 - 4 a c)^2}$$

$$\left(\frac{1}{(a + b x^2 + c x^4)^2} 4 \sqrt{x} (28 a^2 c + a (5 b^2 + 36 b c x^2 - 4 c^2 x^4) + b x^2 (9 b^2 + 37 b c x^2 + 24 c^2 x^4)) + \right.$$

$$\operatorname{RootSum}[a + b \#1^4 + c \#1^8 \&,$$

$$\left. \frac{1}{b \#1^3 + 2 c \#1^7} (-5 b^2 \operatorname{Log}[\sqrt{x} - \#1] - 28 a c \operatorname{Log}[\sqrt{x} - \#1] + 72 b c \operatorname{Log}[\sqrt{x} - \#1] \#1^4) \& \right]$$

Problem 1085: Result is not expressed in closed-form.

$$\int \frac{x^{5/2}}{(a + b x^2 + c x^4)^3} dx$$

Optimal (type 3, 594 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{x^{3/2} (b+2 c x^2)}{4 (b^2-4 a c) (a+b x^2+c x^4)^2} + \frac{3 x^{3/2} (b (b^2+4 a c)+c (b^2+12 a c) x^2)}{16 a (b^2-4 a c)^2 (a+b x^2+c x^4)} + \\
 & \frac{3 c^{1/4} \left(b^2+12 a c - \frac{b^3}{\sqrt{b^2-4 a c}} + \frac{68 a b c}{\sqrt{b^2-4 a c}} \right) \operatorname{ArcTan} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b-\sqrt{b^2-4 a c})^{1/4}} \right]}{32 \times 2^{3/4} a (b^2-4 a c)^2 (-b-\sqrt{b^2-4 a c})^{1/4}} + \\
 & \left(3 c^{1/4} \left(b^3-68 a b c + \sqrt{b^2-4 a c} (b^2+12 a c) \right) \operatorname{ArcTan} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b+\sqrt{b^2-4 a c})^{1/4}} \right] \right) / \\
 & \left(32 \times 2^{3/4} a (b^2-4 a c)^{5/2} (-b+\sqrt{b^2-4 a c})^{1/4} \right) - \\
 & \frac{3 c^{1/4} \left(b^2+12 a c - \frac{b^3}{\sqrt{b^2-4 a c}} + \frac{68 a b c}{\sqrt{b^2-4 a c}} \right) \operatorname{ArcTanh} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b-\sqrt{b^2-4 a c})^{1/4}} \right]}{32 \times 2^{3/4} a (b^2-4 a c)^2 (-b-\sqrt{b^2-4 a c})^{1/4}} - \\
 & \left(3 c^{1/4} \left(b^3-68 a b c + \sqrt{b^2-4 a c} (b^2+12 a c) \right) \operatorname{ArcTanh} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b+\sqrt{b^2-4 a c})^{1/4}} \right] \right) / \\
 & \left(32 \times 2^{3/4} a (b^2-4 a c)^{5/2} (-b+\sqrt{b^2-4 a c})^{1/4} \right)
 \end{aligned}$$

Result (type 7, 222 leaves):

$$\begin{aligned}
 & (-16 a (b^2-4 a c) x^{3/2} (b+2 c x^2) + \\
 & 12 x^{3/2} (b^3+4 a b c+b^2 c x^2+12 a c^2 x^2) (a+b x^2+c x^4) + 3 (a+b x^2+c x^4)^2 \\
 & \operatorname{RootSum} [a+b \#1^4+c \#1^8 \&, \frac{1}{b \#1+2 c \#1^5} (b^3 \operatorname{Log} [\sqrt{x}-\#1]-28 a b c \operatorname{Log} [\sqrt{x}-\#1]+b^2 c \\
 & \operatorname{Log} [\sqrt{x}-\#1] \#1^4+12 a c^2 \operatorname{Log} [\sqrt{x}-\#1] \#1^4) \&]) / (64 a (b^2-4 a c)^2 (a+b x^2+c x^4)^2)
 \end{aligned}$$

Problem 1086: Result is not expressed in closed-form.

$$\int \frac{x^{3/2}}{(a+b x^2+c x^4)^3} dx$$

Optimal (type 3, 594 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{\sqrt{x} (b + 2 c x^2)}{4 (b^2 - 4 a c) (a + b x^2 + c x^4)^2} + \frac{\sqrt{x} (b (b^2 + 20 a c) + c (b^2 + 44 a c) x^2)}{16 a (b^2 - 4 a c)^2 (a + b x^2 + c x^4)} \\
 & \frac{3 c^{3/4} \left(b^2 + 44 a c - \frac{b^3}{\sqrt{b^2 - 4 a c}} + \frac{68 a b c}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTan} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}} \right]}{32 \times 2^{1/4} a (b^2 - 4 a c)^2 (-b - \sqrt{b^2 - 4 a c})^{3/4}} - \\
 & \left(3 c^{3/4} \left(b^3 - 68 a b c + \sqrt{b^2 - 4 a c} (b^2 + 44 a c) \right) \text{ArcTan} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}} \right] \right) / \\
 & \left(32 \times 2^{1/4} a (b^2 - 4 a c)^{5/2} (-b + \sqrt{b^2 - 4 a c})^{3/4} \right) - \\
 & \frac{3 c^{3/4} \left(b^2 + 44 a c - \frac{b^3}{\sqrt{b^2 - 4 a c}} + \frac{68 a b c}{\sqrt{b^2 - 4 a c}} \right) \text{ArcTanh} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}} \right]}{32 \times 2^{1/4} a (b^2 - 4 a c)^2 (-b - \sqrt{b^2 - 4 a c})^{3/4}} - \\
 & \left(3 c^{3/4} \left(b^3 - 68 a b c + \sqrt{b^2 - 4 a c} (b^2 + 44 a c) \right) \text{ArcTanh} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}} \right] \right) / \\
 & \left(32 \times 2^{1/4} a (b^2 - 4 a c)^{5/2} (-b + \sqrt{b^2 - 4 a c})^{3/4} \right)
 \end{aligned}$$

Result (type 7, 224 leaves):

$$\begin{aligned}
 & (-16 a (b^2 - 4 a c) \sqrt{x} (b + 2 c x^2) + \\
 & 4 \sqrt{x} (b^3 + 20 a b c + b^2 c x^2 + 44 a c^2 x^2) (a + b x^2 + c x^4) + 3 (a + b x^2 + c x^4)^2 \\
 & \text{RootSum} [a + b \#1^4 + c \#1^8 \&, \frac{1}{b \#1^3 + 2 c \#1^7} (b^3 \text{Log} [\sqrt{x} - \#1] - 12 a b c \text{Log} [\sqrt{x} - \#1] + b^2 c \\
 & \text{Log} [\sqrt{x} - \#1] \#1^4 + 44 a c^2 \text{Log} [\sqrt{x} - \#1] \#1^4) \&]) / (64 a (b^2 - 4 a c)^2 (a + b x^2 + c x^4)^2)
 \end{aligned}$$

Problem 1087: Result is not expressed in closed-form.

$$\int \frac{\sqrt{x}}{(a + b x^2 + c x^4)^3} dx$$

Optimal (type 3, 658 leaves, 10 steps):

$$\begin{aligned}
 & \frac{x^{3/2} (b^2 - 2 a c + b c x^2)}{4 a (b^2 - 4 a c) (a + b x^2 + c x^4)^2} + \frac{x^{3/2} (5 b^4 - 45 a b^2 c + 52 a^2 c^2 + b c (5 b^2 - 44 a c) x^2)}{16 a^2 (b^2 - 4 a c)^2 (a + b x^2 + c x^4)} - \\
 & \left(c^{1/4} (5 b^4 - 54 a b^2 c + 520 a^2 c^2 - b (5 b^2 - 44 a c) \sqrt{b^2 - 4 a c}) \operatorname{ArcTan} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}} \right] \right) / \\
 & \left(32 \times 2^{3/4} a^2 (b^2 - 4 a c)^{5/2} (-b - \sqrt{b^2 - 4 a c})^{1/4} \right) + \\
 & \left(c^{1/4} (5 b^4 - 54 a b^2 c + 520 a^2 c^2 + b (5 b^2 - 44 a c) \sqrt{b^2 - 4 a c}) \operatorname{ArcTan} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}} \right] \right) / \\
 & \left(32 \times 2^{3/4} a^2 (b^2 - 4 a c)^{5/2} (-b + \sqrt{b^2 - 4 a c})^{1/4} \right) + \\
 & \left(c^{1/4} (5 b^4 - 54 a b^2 c + 520 a^2 c^2 - b (5 b^2 - 44 a c) \sqrt{b^2 - 4 a c}) \operatorname{ArcTanh} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}} \right] \right) / \\
 & \left(32 \times 2^{3/4} a^2 (b^2 - 4 a c)^{5/2} (-b - \sqrt{b^2 - 4 a c})^{1/4} \right) - \\
 & \left(c^{1/4} (5 b^4 - 54 a b^2 c + 520 a^2 c^2 + b (5 b^2 - 44 a c) \sqrt{b^2 - 4 a c}) \operatorname{ArcTanh} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}} \right] \right) / \\
 & \left(32 \times 2^{3/4} a^2 (b^2 - 4 a c)^{5/2} (-b + \sqrt{b^2 - 4 a c})^{1/4} \right)
 \end{aligned}$$

Result (type 7, 254 leaves):

$$\begin{aligned}
 & \frac{1}{64 a^2 (b^2 - 4 a c)^2} \left(- \frac{16 a (-b^2 + 4 a c) x^{3/2} (b^2 - 2 a c + b c x^2)}{(a + b x^2 + c x^4)^2} + \right. \\
 & \left. \frac{4 x^{3/2} (5 b^4 - 45 a b^2 c + 52 a^2 c^2 + 5 b^3 c x^2 - 44 a b c^2 x^2)}{a + b x^2 + c x^4} + \right. \\
 & \left. \operatorname{RootSum} \left[a + b \#1^4 + c \#1^8 \&, \frac{1}{b \#1 + 2 c \#1^5} \left(5 b^4 \operatorname{Log} [\sqrt{x} - \#1] - 49 a b^2 c \operatorname{Log} [\sqrt{x} - \#1] + \right. \right. \right. \\
 & \left. \left. \left. 260 a^2 c^2 \operatorname{Log} [\sqrt{x} - \#1] + 5 b^3 c \operatorname{Log} [\sqrt{x} - \#1] \#1^4 - 44 a b c^2 \operatorname{Log} [\sqrt{x} - \#1] \#1^4 \right) \& \right] \right)
 \end{aligned}$$

Problem 1088: Result is not expressed in closed-form.

$$\int \frac{1}{\sqrt{x} (a + b x^2 + c x^4)^3} dx$$

Optimal (type 3, 658 leaves, 10 steps):

$$\frac{\sqrt{x} (b^2 - 2 a c + b c x^2)}{4 a (b^2 - 4 a c) (a + b x^2 + c x^4)^2} + \frac{\sqrt{x} (7 b^4 - 55 a b^2 c + 60 a^2 c^2 + b c (7 b^2 - 52 a c) x^2)}{16 a^2 (b^2 - 4 a c)^2 (a + b x^2 + c x^4)} +$$

$$\left(3 c^{3/4} (7 b^4 - 66 a b^2 c + 280 a^2 c^2 - b (7 b^2 - 52 a c) \sqrt{b^2 - 4 a c}) \operatorname{ArcTan} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}} \right] \right) /$$

$$\left(32 \times 2^{1/4} a^2 (b^2 - 4 a c)^{5/2} (-b - \sqrt{b^2 - 4 a c})^{3/4} \right) -$$

$$\left(3 c^{3/4} (7 b^4 - 66 a b^2 c + 280 a^2 c^2 + b (7 b^2 - 52 a c) \sqrt{b^2 - 4 a c}) \operatorname{ArcTan} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}} \right] \right) /$$

$$\left(32 \times 2^{1/4} a^2 (b^2 - 4 a c)^{5/2} (-b + \sqrt{b^2 - 4 a c})^{3/4} \right) +$$

$$\left(3 c^{3/4} (7 b^4 - 66 a b^2 c + 280 a^2 c^2 - b (7 b^2 - 52 a c) \sqrt{b^2 - 4 a c}) \operatorname{ArcTanh} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b - \sqrt{b^2 - 4 a c})^{1/4}} \right] \right) /$$

$$\left(32 \times 2^{1/4} a^2 (b^2 - 4 a c)^{5/2} (-b - \sqrt{b^2 - 4 a c})^{3/4} \right) -$$

$$\left(3 c^{3/4} (7 b^4 - 66 a b^2 c + 280 a^2 c^2 + b (7 b^2 - 52 a c) \sqrt{b^2 - 4 a c}) \operatorname{ArcTanh} \left[\frac{2^{1/4} c^{1/4} \sqrt{x}}{(-b + \sqrt{b^2 - 4 a c})^{1/4}} \right] \right) /$$

$$\left(32 \times 2^{1/4} a^2 (b^2 - 4 a c)^{5/2} (-b + \sqrt{b^2 - 4 a c})^{3/4} \right)$$

Result (type 7, 258 leaves):

$$\frac{1}{64 a^2 (b^2 - 4 a c)^2} \left(- \frac{16 a (-b^2 + 4 a c) \sqrt{x} (b^2 - 2 a c + b c x^2)}{(a + b x^2 + c x^4)^2} + \right.$$

$$\frac{4 \sqrt{x} (7 b^4 - 55 a b^2 c + 60 a^2 c^2 + 7 b^3 c x^2 - 52 a b c^2 x^2)}{a + b x^2 + c x^4} +$$

$$3 \operatorname{RootSum} [a + b \#1^4 + c \#1^8 \&, \frac{1}{b \#1^3 + 2 c \#1^7} (7 b^4 \operatorname{Log} [\sqrt{x} - \#1] - 59 a b^2 c \operatorname{Log} [\sqrt{x} - \#1] +$$

$$140 a^2 c^2 \operatorname{Log} [\sqrt{x} - \#1] + 7 b^3 c \operatorname{Log} [\sqrt{x} - \#1] \#1^4 - 52 a b c^2 \operatorname{Log} [\sqrt{x} - \#1] \#1^4) \&] \left. \right)$$

Problem 1089: Result more than twice size of optimal antiderivative.

$$\int (d x)^{3/2} \sqrt{a + b x^2 + c x^4} dx$$

Optimal (type 6, 147 leaves, 2 steps):

$$\left(2 (d x)^{5/2} \sqrt{a+b x^2+c x^4} \operatorname{AppellF1}\left[\frac{5}{4}, -\frac{1}{2}, -\frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(5 d \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right)$$

Result (type 6, 1048 leaves):

$$\frac{1}{225 c^2 (a+b x^2+c x^4)^{3/2}}$$

$$d \sqrt{d x} \left(10 c (2 b+5 c x^2) (a+b x^2+c x^4)^2 - \left(25 a^2 b \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \right. \right.$$

$$\left. \left. \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \right.$$

$$\left(5 a \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right.$$

$$x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$\left. \left. \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) +$$

$$\left(90 a^2 c x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(9 a \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right.$$

$$x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$\left. \left. \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) +$$

$$\left(27 a b^2 x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2 \right) \left(b+\sqrt{b^2-4 a c}+2 c x^2 \right) \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(-9 a \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$x^2 \left(\left(b+\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$\left. \left. \left(b-\sqrt{b^2-4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \right)$$

Problem 1090: Result more than twice size of optimal antiderivative.

$$\int \sqrt{d x} \sqrt{a+b x^2+c x^4} d x$$

Optimal (type 6, 147 leaves, 2 steps):

$$\left(2 (d x)^{3/2} \sqrt{a+b x^2+c x^4} \operatorname{AppellF1}\left[\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] \right) / \left(3 d \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right)$$

Result (type 6, 706 leaves):

$$\frac{1}{147 (a+b x^2+c x^4)^{3/2}} 2 x \sqrt{d x} \left(21 (a+b x^2+c x^4)^2 + \left(49 a^2 (b-\sqrt{b^2-4 a c}+2 c x^2) (b+\sqrt{b^2-4 a c}+2 c x^2) \operatorname{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \left(c \left(7 a \operatorname{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - x^2 \left((b+\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + (b-\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \right) + \left(33 a b x^2 (b-\sqrt{b^2-4 a c}+2 c x^2) (b+\sqrt{b^2-4 a c}+2 c x^2) \operatorname{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \left(22 a c \operatorname{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - 2 c x^2 \left((b+\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{11}{4},\frac{1}{2},\frac{3}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + (b-\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{11}{4},\frac{3}{2},\frac{1}{2},\frac{15}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},-\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \right)$$

Problem 1091: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b x^2+c x^4}}{\sqrt{d x}} d x$$

Optimal (type 6, 145 leaves, 2 steps):

$$\left(2 \sqrt{d x} \sqrt{a + b x^2 + c x^4} \operatorname{AppellF1}\left[\frac{1}{4}, -\frac{1}{2}, -\frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right] \right) /$$

$$\left(d \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \right)$$

Result (type 6, 709 leaves):

$$\frac{1}{25 \sqrt{d x} (a + b x^2 + c x^4)^{3/2}}$$

$$2 x \left(5 (a + b x^2 + c x^4)^2 + \left(25 a^2 (b - \sqrt{b^2 - 4 a c} + 2 c x^2) (b + \sqrt{b^2 - 4 a c} + 2 c x^2) \right. \right.$$

$$\left. \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) / \right.$$

$$\left(c \left(5 a \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] - \right. \right.$$

$$x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] + \right.$$

$$\left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) \right) \right) +$$

$$\left(9 a b x^2 (b - \sqrt{b^2 - 4 a c} + 2 c x^2) (b + \sqrt{b^2 - 4 a c} + 2 c x^2) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \right. \right.$$

$$\left. \left. \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) /$$

$$\left(2 c \left(9 a \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] - \right. \right.$$

$$x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] + \right.$$

$$\left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) \right) \right) \right)$$

Problem 1092: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + b x^2 + c x^4}}{(d x)^{3/2}} dx$$

Optimal (type 6, 145 leaves, 2 steps):

$$- \left(2 \sqrt{a+bx^2+cx^4} \operatorname{AppellF1} \left[-\frac{1}{4}, -\frac{1}{2}, -\frac{1}{2}, \frac{3}{4}, -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}} \right] \right) / \left(d \sqrt{dx} \sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}} \right)$$

Result (type 6, 707 leaves):

$$\frac{1}{21 (dx)^{3/2} (a+bx^2+cx^4)^{3/2}} 2x \left(-21 (a+bx^2+cx^4)^2 + \left(49abx^2 (b-\sqrt{b^2-4ac}+2cx^2) (b+\sqrt{b^2-4ac}+2cx^2) \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) / \left(2c \left(7a \operatorname{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] - x^2 \left((b+\sqrt{b^2-4ac}) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + (b-\sqrt{b^2-4ac}) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) \right) \right) + \left(33ax^4 (b-\sqrt{b^2-4ac}+2cx^2) (b+\sqrt{b^2-4ac}+2cx^2) \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) / \left(11a \operatorname{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] - x^2 \left((b+\sqrt{b^2-4ac}) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + (b-\sqrt{b^2-4ac}) \operatorname{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) \right) \right)$$

Problem 1093: Result more than twice size of optimal antiderivative.

$$\int (dx)^{3/2} (a+bx^2+cx^4)^{3/2} dx$$

Optimal (type 6, 148 leaves, 2 steps):

$$\left(2a (dx)^{5/2} \sqrt{a+bx^2+cx^4} \operatorname{AppellF1} \left[\frac{5}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{9}{4}, -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}} \right] \right) / \left(5d \sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}} \right)$$

Result (type 6, 1751 leaves):

$$\begin{aligned}
 & \frac{1}{16575 c^3 (a+b x^2+c x^4)^{3/2}} \\
 & 2 d \sqrt{d x} \left(5 c (a+b x^2+c x^4)^2 (-28 b^3+20 b^2 c x^2+65 c^2 x^2 (7 a+3 c x^4))+b c (176 a+285 c x^4) \right) + \\
 & \left(175 a^2 b^3 (b-\sqrt{b^2-4 a c}+2 c x^2) (b+\sqrt{b^2-4 a c}+2 c x^2) \right. \\
 & \quad \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \\
 & \left(5 a \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right. \\
 & \quad x^2 \left((b+\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\
 & \quad \left. (b-\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \left. \right) - \\
 & \left(1100 a^3 b c (b-\sqrt{b^2-4 a c}+2 c x^2) (b+\sqrt{b^2-4 a c}+2 c x^2) \right. \\
 & \quad \left. \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \\
 & \left(5 a \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right. \\
 & \quad x^2 \left((b+\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\
 & \quad \left. (b-\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \left. \right) + \\
 & \left(189 a b^4 x^2 (b-\sqrt{b^2-4 a c}+2 c x^2) (b+\sqrt{b^2-4 a c}+2 c x^2) \right. \\
 & \quad \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \\
 & \left(9 a \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right. \\
 & \quad x^2 \left((b+\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\
 & \quad \left. (b-\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \left. \right) + \\
 & \left(2340 a^3 c^2 x^2 (b-\sqrt{b^2-4 a c}+2 c x^2) (b+\sqrt{b^2-4 a c}+2 c x^2) \right. \\
 & \quad \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) /
 \end{aligned}$$

$$\begin{aligned} & \left(9 a \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right. \\ & x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & \left. \left. \left(b-\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) + \\ & \left(1413 a^2 b^2 c x^2 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \right. \\ & \left. \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \\ & \left(-9 a \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & \left. \left. \left(b-\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) \end{aligned}$$

Problem 1094: Result more than twice size of optimal antiderivative.

$$\int \sqrt{d x} (a+b x^2+c x^4)^{3 / 2} d x$$

Optimal (type 6, 148 leaves, 2 steps):

$$\begin{aligned} & \left(2 a (d x)^{3 / 2} \sqrt{a+b x^2+c x^4} \operatorname{AppellF1}\left[\frac{3}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{7}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] \right) / \\ & \left(3 d \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right) \end{aligned}$$

Result (type 6, 1395 leaves):

$$\begin{aligned} & \frac{1}{8085 c^2 (a+b x^2+c x^4)^{3 / 2}} 2 x \sqrt{d x} \\ & \left(7 c (a+b x^2+c x^4)^2 (12 b^2+119 b c x^2+11 c (19 a+7 c x^4)) - (147 a^2 b^2 (b-\sqrt{b^2-4 a c}+2 c x^2) \right. \\ & \left. (b+\sqrt{b^2-4 a c}+2 c x^2) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \\ & \left(7 a \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right. \\ & x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\ & \left. \left. \left(b-\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) + \end{aligned}$$

$$\begin{aligned}
 & \left(2156 a^3 c \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(7 a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] - \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\
 & \left(1188 a^2 b c x^2 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(11 a \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] - \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\
 & \left(165 a b^3 x^2 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(-11 a \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) \right)
 \end{aligned}$$

Problem 1095: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^2 + c x^4)^{3/2}}{\sqrt{d x}} dx$$

Optimal (type 6, 146 leaves, 2 steps):

$$\left(2 a \sqrt{d x} \sqrt{a+b x^2+c x^4} \operatorname{AppellF1}\left[\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{5}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(d \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}} \right)$$

Result (type 6, 1395 leaves):

$$\frac{1}{975 c^2 \sqrt{d x} (a+b x^2+c x^4)^{3/2}}$$

$$2 x \left(5 c (a+b x^2+c x^4)^2 (4 b^2+25 b c x^2+3 c (17 a+5 c x^4)) - \left(25 a^2 b^2 (b-\sqrt{b^2-4 a c}+2 c x^2) \right. \right.$$

$$\left. \left. (b+\sqrt{b^2-4 a c}+2 c x^2) \operatorname{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) / \right.$$

$$\left(5 a \operatorname{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right.$$

$$x^2 \left((b+\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$\left. \left. (b-\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) +$$

$$\left(900 a^3 c (b-\sqrt{b^2-4 a c}+2 c x^2) (b+\sqrt{b^2-4 a c}+2 c x^2) \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(5 a \operatorname{AppellF1}\left[\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right.$$

$$x^2 \left((b+\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$\left. \left. (b-\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{5}{4},\frac{3}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) +$$

$$\left(252 a^2 b c x^2 (b-\sqrt{b^2-4 a c}+2 c x^2) (b+\sqrt{b^2-4 a c}+2 c x^2) \right.$$

$$\left. \operatorname{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) /$$

$$\left(9 a \operatorname{AppellF1}\left[\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] - \right.$$

$$x^2 \left((b+\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{9}{4},\frac{1}{2},\frac{3}{2},\frac{13}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right.$$

$$\left. \left. (b-\sqrt{b^2-4 a c}) \operatorname{AppellF1}\left[\frac{9}{4},\frac{3}{2},\frac{1}{2},\frac{13}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] \right) \right) +$$

$$\begin{aligned}
 & \left(27 a b^3 x^2 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(-9 a \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \Big)
 \end{aligned}$$

Problem 1096: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^2 + c x^4)^{3/2}}{(d x)^{3/2}} dx$$

Optimal (type 6, 146 leaves, 2 steps):

$$\begin{aligned}
 & - \left(\left(2 a \sqrt{a + b x^2 + c x^4} \text{AppellF1} \left[-\frac{1}{4}, -\frac{3}{2}, -\frac{3}{2}, \frac{3}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}} \right] \right) / \right. \\
 & \quad \left. \left(d \sqrt{d x} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \right) \right)
 \end{aligned}$$

Result (type 6, 1059 leaves):

$$\begin{aligned}
 & \frac{1}{539 (dx)^{3/2} (a + bx^2 + cx^4)^{3/2}} \\
 & 2x \left(7 (a + bx^2 + cx^4)^2 (-77a + 13bx^2 + 7cx^4) + \left(784a^2bx^2 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \right. \\
 & \quad \left. \left. \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \right. \\
 & \quad \left(c \left(7a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) + \\
 & \quad \left(924a^2x^4 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \right. \right. \\
 & \quad \quad \left. \left. \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \right. \\
 & \quad \left(11a \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) + \\
 & \quad \left(33ab^2x^4 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \quad \left. \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \right. \\
 & \quad \left(c \left(11a \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \right. \\
 & \quad \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \quad \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right) \right) \right)
 \end{aligned}$$

Problem 1097: Result more than twice size of optimal antiderivative.

$$\int \frac{(dx)^{3/2}}{\sqrt{a + bx^2 + cx^4}} dx$$

Optimal (type 6, 147 leaves, 2 steps):

$$\left(2 (dx)^{5/2} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \right. \\ \left. \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}\right] \right) / \left(5d \sqrt{a + bx^2 + cx^4} \right)$$

Result (type 6, 386 leaves):

$$- \left(\left(18a^2 x (dx)^{3/2} \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \right. \\ \left. \left. \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) / \right. \\ \left(5 \left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) \left(a + bx^2 + cx^4 \right)^{3/2} \right. \\ \left(-9a \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] + \right. \\ \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] + \right. \right. \\ \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) \right) \right) \right)$$

Problem 1098: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{dx}}{\sqrt{a + bx^2 + cx^4}} dx$$

Optimal (type 6, 147 leaves, 2 steps):

$$\left(2 (dx)^{3/2} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \right. \\ \left. \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}\right] \right) / \left(3d \sqrt{a + bx^2 + cx^4} \right)$$

Result (type 6, 386 leaves):

$$\begin{aligned}
 & - \left(\left(14 a^2 x \sqrt{d x} \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \right. \\
 & \quad \left. \left. \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \right. \\
 & \quad \left(3 \left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left. \left(-7 a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \right. \\
 & \quad \left. \left. x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \right. \right. \\
 & \quad \left. \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) \right)
 \end{aligned}$$

Problem 1099: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{d x} \sqrt{a + b x^2 + c x^4}} dx$$

Optimal (type 6, 145 leaves, 2 steps):

$$\begin{aligned}
 & \frac{1}{d \sqrt{a + b x^2 + c x^4}} 2 \sqrt{d x} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \\
 & \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}} \right]
 \end{aligned}$$

Result (type 6, 384 leaves):

$$\begin{aligned}
 & - \left(\left(10 a^2 x \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \right. \\
 & \quad \left. \left. \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \right. \\
 & \quad \left(\left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) \sqrt{d x} \left(a + b x^2 + c x^4 \right)^{3/2} \right. \\
 & \quad \left. \left(-5 a \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \right. \\
 & \quad \left. \left. x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \right. \right. \\
 & \quad \left. \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) \right)
 \end{aligned}$$

Problem 1100: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(dx)^{3/2} \sqrt{a+bx^2+cx^4}} dx$$

Optimal (type 6, 145 leaves, 2 steps):

$$-\left(2 \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \operatorname{AppellF1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}\right] \right) / \left(d \sqrt{dx} \sqrt{a+bx^2+cx^4}\right)$$

Result (type 6, 710 leaves):

$$\frac{1}{21 a (dx)^{3/2} (a+bx^2+cx^4)^{3/2}} 2x \left(-21 (a+bx^2+cx^4)^2 + \left(49 a b x^2 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) / \left(4c \left(7a \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] - x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] + \left(b - \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) \right) \right) + \left(99 a x^4 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) / \left(44 a \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] - 4x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] + \left(b - \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) \right) \right)$$

Problem 1101: Result more than twice size of optimal antiderivative.

$$\int \frac{(dx)^{3/2}}{(a+bx^2+cx^4)^{3/2}} dx$$

Optimal (type 6, 150 leaves, 2 steps):

$$\left(2 (d x)^{5/2} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right] \right) / \left(5 a d \sqrt{a + b x^2 + c x^4} \right)$$

Result (type 6, 720 leaves):

$$\frac{1}{5 (b^2 - 4 a c) (a + b x^2 + c x^4)^{3/2}} d \sqrt{d x} \left(-5 (b + 2 c x^2) (a + b x^2 + c x^4) + \left(25 a b \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) / \left(4 c \left(5 a \operatorname{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] - x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] + \left(b - \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) \right) \right) + \left(9 a x^2 \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) / \left(18 a \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] - 2 x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] + \left(b - \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) \right) \right)$$

Problem 1102: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{d x}}{(a + b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 6, 150 leaves, 2 steps):

$$\left(2 (d x)^{3/2} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \right. \\ \left. \text{AppellF1}\left[\frac{3}{4}, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right] \right) / \left(3 a d \sqrt{a + b x^2 + c x^4} \right)$$

Result (type 6, 1058 leaves):

$$\frac{1}{84 a (-b^2 + 4 a c) (a + b x^2 + c x^4)^{3/2}} \\ x \sqrt{d x} \left(-84 (b^2 - 2 a c + b c x^2) (a + b x^2 + c x^4) + \left(196 a^2 (b - \sqrt{b^2 - 4 a c} + 2 c x^2) \right. \right. \\ \left. \left. (b + \sqrt{b^2 - 4 a c} + 2 c x^2) \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) / \right. \\ \left(14 a \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] - \right. \\ \left. 2 x^2 \left((b + \sqrt{b^2 - 4 a c}) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] + \right. \right. \\ \left. \left. (b - \sqrt{b^2 - 4 a c}) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) \right) + \\ \left(49 a b^2 (b - \sqrt{b^2 - 4 a c} + 2 c x^2) (b + \sqrt{b^2 - 4 a c} + 2 c x^2) \right. \\ \left. \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) / \\ \left(c \left(7 a \text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] - \right. \right. \\ \left. \left. x^2 \left((b + \sqrt{b^2 - 4 a c}) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] + \right. \right. \right. \\ \left. \left. \left. (b - \sqrt{b^2 - 4 a c}) \text{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) \right) \right) \right) - \\ \left(99 a b x^2 (b - \sqrt{b^2 - 4 a c} + 2 c x^2) (b + \sqrt{b^2 - 4 a c} + 2 c x^2) \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \right. \right. \\ \left. \left. \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) / \\ \left(-11 a \text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] + \right. \\ \left. x^2 \left((b + \sqrt{b^2 - 4 a c}) \text{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] + \right. \right. \\ \left. \left. (b - \sqrt{b^2 - 4 a c}) \text{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) \right) \right) \right)$$

Problem 1103: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{d x} (a + b x^2 + c x^4)^{3/2}} dx$$

Optimal (type 6, 148 leaves, 2 steps):

$$\left(2 \sqrt{d x} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \operatorname{AppellF1}\left[\frac{1}{4}, \frac{3}{2}, \frac{3}{2}, \frac{5}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right] \right) / \left(a d \sqrt{a + b x^2 + c x^4} \right)$$

Result (type 6, 1058 leaves):

$$\begin{aligned}
 & \frac{1}{20a(-b^2+4ac)\sqrt{dx}(a+bx^2+cx^4)^{3/2}} \\
 & x \left(-20(b^2-2ac+bcx^2)(a+bx^2+cx^4) + \left(300a^2 \left(b - \sqrt{b^2-4ac} + 2cx^2 \right) \right. \right. \\
 & \quad \left. \left. \left(b + \sqrt{b^2-4ac} + 2cx^2 \right) \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) \right) / \\
 & \left(10a \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] - \right. \\
 & \quad 2x^2 \left(\left(b + \sqrt{b^2-4ac} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2-4ac} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) \right) - \\
 & \left(25ab^2 \left(b - \sqrt{b^2-4ac} + 2cx^2 \right) \left(b + \sqrt{b^2-4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) / \\
 & \left(c \left(5a \text{AppellF1} \left[\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] - \right. \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2-4ac} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{3}{2}, \frac{9}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2-4ac} \right) \text{AppellF1} \left[\frac{5}{4}, \frac{3}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) \right) \right) - \\
 & \left(9abx^2 \left(b - \sqrt{b^2-4ac} + 2cx^2 \right) \left(b + \sqrt{b^2-4ac} + 2cx^2 \right) \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \right. \right. \\
 & \quad \left. \left. \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) / \\
 & \left(-9a \text{AppellF1} \left[\frac{5}{4}, \frac{1}{2}, \frac{1}{2}, \frac{9}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2-4ac} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{1}{2}, \frac{3}{2}, \frac{13}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] + \right. \\
 & \quad \left. \left. \left(b - \sqrt{b^2-4ac} \right) \text{AppellF1} \left[\frac{9}{4}, \frac{3}{2}, \frac{1}{2}, \frac{13}{4}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}} \right] \right) \right) \right)
 \end{aligned}$$

Problem 1104: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(dx)^{3/2} (a+bx^2+cx^4)^{3/2}} dx$$

Optimal (type 6, 148 leaves, 2 steps):

$$- \left(\left(2 \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \right. \right. \\ \left. \left. \text{AppellF1} \left[-\frac{1}{4}, \frac{3}{2}, \frac{3}{2}, \frac{3}{4}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}} \right] \right) / \left(a d \sqrt{d x} \sqrt{a + b x^2 + c x^4} \right) \right)$$

Result (type 6, 1600 leaves):

$$\frac{1}{7 (d x)^{3/2} (a + b x^2 + c x^4)^{3/2}} \\ \times \left(\frac{7 x^2 (b^3 - 3 a b c + b^2 c x^2 - 2 a c^2 x^2) (a + b x^2 + c x^4)}{a^2 (-b^2 + 4 a c)} - \frac{14 (a + b x^2 + c x^4)^2}{a^2} + \right. \\ \left(49 b^3 x^2 (b - \sqrt{b^2 - 4 a c} + 2 c x^2) (b + \sqrt{b^2 - 4 a c} + 2 c x^2) \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \right. \right. \\ \left. \left. \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \left((b^2 - 4 a c) (-b + \sqrt{b^2 - 4 a c}) \right) \\ \left(b + \sqrt{b^2 - 4 a c} \right) \left(-7 a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\ \left. x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \right. \\ \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) - \\ \left(147 a b c x^2 (b - \sqrt{b^2 - 4 a c} + 2 c x^2) (b + \sqrt{b^2 - 4 a c} + 2 c x^2) \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \right. \right. \\ \left. \left. \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \left((b^2 - 4 a c) (-b + \sqrt{b^2 - 4 a c}) \right) \\ \left(b + \sqrt{b^2 - 4 a c} \right) \left(-7 a \text{AppellF1} \left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\ \left. x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \right. \\ \left. \left. \left(b - \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\ \left(99 b^2 c x^4 (b - \sqrt{b^2 - 4 a c} + 2 c x^2) (b + \sqrt{b^2 - 4 a c} + 2 c x^2) \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \right. \right. \\ \left. \left. \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \left((b^2 - 4 a c) (-b + \sqrt{b^2 - 4 a c}) \right) \\ \left(b + \sqrt{b^2 - 4 a c} \right) \left(-11 a \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\ \left. x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \right.$$

$$\begin{aligned}
 & \left(\left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \Bigg) - \\
 & \left(330ac^2x^4 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \left. \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \left((b^2 - 4ac) \left(-b + \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) \right) \\
 & \left(-11a \text{AppellF1} \left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \left. x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \right. \\
 & \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \Bigg) \Bigg)
 \end{aligned}$$

Problem 1108: Result is not expressed in closed-form.

$$\int \frac{(dx)^m}{a + bx^2 + cx^4} dx$$

Optimal (type 5, 173 leaves, 3 steps):

$$\frac{2c(dx)^{1+m} \text{Hypergeometric2F1} \left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}} \right]}{\sqrt{b^2 - 4ac} \left(b - \sqrt{b^2 - 4ac} \right) d(1+m)} - \frac{2c(dx)^{1+m} \text{Hypergeometric2F1} \left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}} \right]}{\sqrt{b^2 - 4ac} \left(b + \sqrt{b^2 - 4ac} \right) d(1+m)}$$

Result (type 7, 82 leaves):

$$\frac{1}{2m} (dx)^m \text{RootSum} \left[a + b\#1^2 + c\#1^4 \&, \frac{\text{Hypergeometric2F1} \left[-m, -m, 1-m, -\frac{\#1}{x-\#1} \right] \left(\frac{x}{x-\#1} \right)^{-m}}{b\#1 + 2c\#1^3} \& \right]$$

Problem 1109: Result unnecessarily involves higher level functions.

$$\int \frac{(dx)^m}{(a + bx^2 + cx^4)^2} dx$$

Optimal (type 5, 315 leaves, 4 steps):

$$\frac{(dx)^{1+m} (b^2 - 2ac + bcx^2)}{2a(b^2 - 4ac)d(a+bx^2+cx^4)} +$$

$$\left(c \left(b^2(1-m) + b\sqrt{b^2 - 4ac}(1-m) - 4ac(3-m) \right) (dx)^{1+m} \text{Hypergeometric2F1} \left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}} \right] \right) / \left(2a(b^2 - 4ac)^{3/2} (b - \sqrt{b^2 - 4ac}) d(1+m) \right) -$$

$$\left(c \left(b^2(1-m) - b\sqrt{b^2 - 4ac}(1-m) - 4ac(3-m) \right) (dx)^{1+m} \text{Hypergeometric2F1} \left[1, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}} \right] \right) / \left(2a(b^2 - 4ac)^{3/2} (b + \sqrt{b^2 - 4ac}) d(1+m) \right)$$

Result (type 6, 376 leaves):

$$\left(a(3+m)x(dx)^m \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \text{AppellF1} \left[\frac{1+m}{2}, 2, 2, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \left(4c(1+m) \right)$$

$$(a+bx^2+cx^4)^3 \left(a(3+m) \text{AppellF1} \left[\frac{1+m}{2}, 2, 2, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right.$$

$$2x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{3+m}{2}, 2, 3, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right.$$

$$\left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{3+m}{2}, 3, 2, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right)$$

Problem 1110: Result more than twice size of optimal antiderivative.

$$\int (dx)^m (a+bx^2+cx^4)^{3/2} dx$$

Optimal (type 6, 158 leaves, 2 steps):

$$\left(a(dx)^{1+m} \sqrt{a+bx^2+cx^4} \text{AppellF1} \left[\frac{1+m}{2}, -\frac{3}{2}, -\frac{3}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}} \right] \right) /$$

$$\left(d(1+m) \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \right)$$

Result (type 6, 1080 leaves):

$$\begin{aligned}
 & \frac{1}{8 c^2 \sqrt{a+b x^2+c x^4}} \\
 & \left(\left(b - \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} \right) x (d x)^m \left(b - \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \left. \left(\left(a (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{3+m}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \right. \right. \\
 & \left. \left((1+m) \left(2 a (3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{3+m}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) + \right. \right. \\
 & \left. x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{5+m}{2}, \right. \right. \right. \\
 & \left. \left. -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \left(b - \sqrt{b^2 - 4 a c} \right) \right. \\
 & \left. \left. \left. \operatorname{AppellF1} \left[\frac{3+m}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\
 & \left(b (5+m) x^2 \operatorname{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left((3+m) \left(2 a (5+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) + \right. \\
 & \left. x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{5+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{7+m}{2}, \right. \right. \right. \\
 & \left. \left. -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \left(b - \sqrt{b^2 - 4 a c} \right) \right. \\
 & \left. \left. \left. \operatorname{AppellF1} \left[\frac{5+m}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) + \\
 & \left(c (7+m) x^4 \operatorname{AppellF1} \left[\frac{5+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left((5+m) \left(2 a (7+m) \operatorname{AppellF1} \left[\frac{5+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{7+m}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) + \right. \\
 & \left. x^2 \left(\left(b + \sqrt{b^2 - 4 a c} \right) \operatorname{AppellF1} \left[\frac{7+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{9+m}{2}, \right. \right. \right. \\
 & \left. \left. -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \left(b - \sqrt{b^2 - 4 a c} \right) \right. \\
 & \left. \left. \left. \operatorname{AppellF1} \left[\frac{7+m}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{9+m}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) \right)
 \end{aligned}$$

Problem 1111: Result more than twice size of optimal antiderivative.

$$\int (d x)^m \sqrt{a+b x^2+c x^4} d x$$

Optimal (type 6, 157 leaves, 2 steps):

$$\left((dx)^{1+m} \sqrt{a+bx^2+cx^4} \operatorname{AppellF1}\left[\frac{1+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right] \right) / \left(d(1+m) \sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}} \right)$$

Result (type 6, 423 leaves):

$$\left((b-\sqrt{b^2-4ac}) (b+\sqrt{b^2-4ac}) (3+m) x (dx)^m \operatorname{AppellF1}\left[\frac{1+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] \right) / \left(8c^2(1+m) \sqrt{a+bx^2+cx^4} \right) + \left(2a(3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] + x^2 \left((b+\sqrt{b^2-4ac}) \operatorname{AppellF1}\left[\frac{3+m}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] + (b-\sqrt{b^2-4ac}) \operatorname{AppellF1}\left[\frac{3+m}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right] \right) \right)$$

Problem 1112: Result more than twice size of optimal antiderivative.

$$\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}} dx$$

Optimal (type 6, 157 leaves, 2 steps):

$$\left((dx)^{1+m} \sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}} \sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}} \operatorname{AppellF1}\left[\frac{1+m}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right] \right) / \left(d(1+m) \sqrt{a+bx^2+cx^4} \right)$$

Result (type 6, 425 leaves):

$$\begin{aligned}
 & - \left(\left(2a^2 (3+m) x (dx)^m \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \right. \\
 & \quad \left. \left. \text{AppellF1} \left[\frac{1+m}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \right. \\
 & \quad \left(\left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) (1+m) (a + bx^2 + cx^4)^{3/2} \right. \\
 & \quad \left(-2a (3+m) \text{AppellF1} \left[\frac{1+m}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{3+m}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{3+m}{2}, \frac{3}{2}, \frac{1}{2}, \right. \right. \\
 & \quad \quad \left. \left. \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \left. \right) \left. \right) \left. \right)
 \end{aligned}$$

Problem 1113: Result more than twice size of optimal antiderivative.

$$\int \frac{(dx)^m}{(ax^2 + cx^4)^{3/2}} dx$$

Optimal (type 6, 160 leaves, 2 steps):

$$\left((dx)^{1+m} \sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}} \sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}} \text{AppellF1} \left[\frac{1+m}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}} \right] \right) / \left(ad (1+m) \sqrt{a + bx^2 + cx^4} \right)$$

Result (type 6, 426 leaves):

$$\begin{aligned}
 & \left(2a^2 (3+m) x (dx)^m \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\
 & \quad \left. \text{AppellF1} \left[\frac{1+m}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\
 & \quad \left(\left(b - \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} \right) (1+m) (a + bx^2 + cx^4)^{5/2} \right. \\
 & \quad \left(2a (3+m) \text{AppellF1} \left[\frac{1+m}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \\
 & \quad 3x^2 \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{3+m}{2}, \frac{3}{2}, \frac{5}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\
 & \quad \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[\frac{3+m}{2}, \frac{5}{2}, \frac{3}{2}, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \left. \right) \left. \right)
 \end{aligned}$$

Problem 1114: Result more than twice size of optimal antiderivative.

$$\int (dx)^m (a + bx^2 + cx^4)^p dx$$

Optimal (type 6, 155 leaves, 2 steps):

$$\frac{1}{d(1+m)} (dx)^{1+m} \left(1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}\right)^{-p} \left(1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}\right)^{-p} (a + bx^2 + cx^4)^p \text{AppellF1}\left[\frac{1+m}{2}, -p, -p, \frac{3+m}{2}, -\frac{2cx^2}{b - \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}\right]$$

Result (type 6, 499 leaves):

$$\begin{aligned} & - \left(\left(2^{-2-p} c (b + \sqrt{b^2 - 4ac}) (3+m) x (dx)^m \left(\frac{b - \sqrt{b^2 - 4ac}}{2c} + x^2 \right)^{-p} \right. \right. \\ & \left. \left(\frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{c} \right)^{1+p} \left(-2a + (-b + \sqrt{b^2 - 4ac}) x^2 \right)^2 (a + bx^2 + cx^4)^{-1+p} \right. \\ & \left. \text{AppellF1}\left[\frac{1+m}{2}, -p, -p, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) / \\ & \left((-b + \sqrt{b^2 - 4ac}) (1+m) (b + \sqrt{b^2 - 4ac} + 2cx^2) \right. \\ & \left. \left(a(3+m) \text{AppellF1}\left[\frac{1+m}{2}, -p, -p, \frac{3+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] + \right. \right. \\ & \left. \left. px^2 \left((b - \sqrt{b^2 - 4ac}) \text{AppellF1}\left[\frac{3+m}{2}, 1-p, -p, \frac{5+m}{2}, \right. \right. \right. \right. \\ & \left. \left. \left. -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] + (b + \sqrt{b^2 - 4ac}) \right. \right. \\ & \left. \left. \left. \left. \left. \left. \text{AppellF1}\left[\frac{3+m}{2}, -p, 1-p, \frac{5+m}{2}, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, -\frac{2cx^2}{-b + \sqrt{b^2 - 4ac}}\right] \right) \right) \right) \right) \right) \end{aligned}$$

Problem 1115: Result unnecessarily involves higher level functions.

$$\int x^7 (a + bx^2 + cx^4)^p dx$$

Optimal (type 5, 257 leaves, 4 steps):

$$\frac{x^4 (a+b x^2+c x^4)^{1+p}}{4 c (2+p)} +$$

$$\left(\frac{(b^2 (2+p) (3+p) - 2 a c (3+2 p) - 2 b c (1+p) (3+p) x^2) (a+b x^2+c x^4)^{1+p}}{(8 c^3 (1+p) (2+p) (3+2 p))} - \right.$$

$$\left. \left(2^{-2+p} b (6 a c - b^2 (3+p)) \left(-\frac{b - \sqrt{b^2 - 4 a c} + 2 c x^2}{\sqrt{b^2 - 4 a c}} \right)^{-1-p} (a+b x^2+c x^4)^{1+p} \text{Hypergeometric2F1} \left[\right. \right.$$

$$\left. \left. -p, 1+p, 2+p, \frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{2 \sqrt{b^2 - 4 a c}} \right] \right) / \left(c^3 \sqrt{b^2 - 4 a c} (1+p) (3+2 p) \right)$$

Result (type 6, 440 leaves):

$$\left(5 \times 2^{-4+p} c (b + \sqrt{b^2 - 4 a c}) x^8 \left(\frac{b - \sqrt{b^2 - 4 a c}}{2 c} + x^2 \right)^{-p} \right.$$

$$\left. \left(\frac{b - \sqrt{b^2 - 4 a c} + 2 c x^2}{c} \right)^{1+p} \left(2 a + (b - \sqrt{b^2 - 4 a c}) x^2 \right)^2 (a + x^2 (b + c x^2))^{-1+p} \right.$$

$$\left. \text{AppellF1} \left[4, -p, -p, 5, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) /$$

$$\left((-b + \sqrt{b^2 - 4 a c}) (b + \sqrt{b^2 - 4 a c} + 2 c x^2) \right.$$

$$\left. \left(-10 a \text{AppellF1} \left[4, -p, -p, 5, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \right.$$

$$\left. p x^2 \left((-b + \sqrt{b^2 - 4 a c}) \text{AppellF1} \left[5, 1-p, -p, 6, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] - \right. \right.$$

$$\left. \left. (b + \sqrt{b^2 - 4 a c}) \text{AppellF1} \left[5, -p, 1-p, 6, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right)$$

Problem 1116: Result unnecessarily involves higher level functions.

$$\int x^5 (a+b x^2+c x^4)^p dx$$

Optimal (type 5, 223 leaves, 4 steps):

$$-\frac{b (2+p) (a+b x^2+c x^4)^{1+p}}{4 c^2 (1+p) (3+2 p)} + \frac{x^2 (a+b x^2+c x^4)^{1+p}}{2 c (3+2 p)} +$$

$$\left(2^{-1+p} (2 a c - b^2 (2+p)) \left(-\frac{b - \sqrt{b^2 - 4 a c} + 2 c x^2}{\sqrt{b^2 - 4 a c}} \right)^{-1-p} (a+b x^2+c x^4)^{1+p} \text{Hypergeometric2F1} \left[\right. \right.$$

$$\left. \left. -p, 1+p, 2+p, \frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{2 \sqrt{b^2 - 4 a c}} \right] \right) / \left(c^2 \sqrt{b^2 - 4 a c} (1+p) (3+2 p) \right)$$

Result (type 6, 395 leaves):

$$\left(\left(b + \sqrt{b^2 - 4ac} \right) x^6 \left(b - \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(2a + \left(b - \sqrt{b^2 - 4ac} \right) x^2 \right)^2 \right. \\ \left. (a + x^2 (b + cx^2))^{-1+p} \operatorname{AppellF1} \left[3, -p, -p, 4, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \\ \left(3 \left(-b + \sqrt{b^2 - 4ac} \right) \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\ \left(-8a \operatorname{AppellF1} \left[3, -p, -p, 4, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\ \left. px^2 \left(\left(-b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[4, 1-p, -p, 5, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \right. \\ \left. \left. \left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[4, -p, 1-p, 5, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right)$$

Problem 1117: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3 (a + bx^2 + cx^4)^p dx$$

Optimal (type 5, 160 leaves, 3 steps):

$$\frac{(a + bx^2 + cx^4)^{1+p}}{4c(1+p)} + \left(2^{-1+p} b \left(-\frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{\sqrt{b^2 - 4ac}} \right)^{-1-p} (a + bx^2 + cx^4)^{1+p} \right. \\ \left. \operatorname{Hypergeometric2F1} \left[-p, 1+p, 2+p, \frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{2\sqrt{b^2 - 4ac}} \right] \right) / \left(c\sqrt{b^2 - 4ac} (1+p) \right)$$

Result (type 6, 440 leaves):

$$\left(3 \times 2^{-3-p} c \left(b + \sqrt{b^2 - 4ac} \right) x^4 \left(\frac{b - \sqrt{b^2 - 4ac}}{2c} + x^2 \right)^{-p} \right. \\ \left. \left(\frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{c} \right)^{1+p} \left(2a + \left(b - \sqrt{b^2 - 4ac} \right) x^2 \right)^2 (a + x^2 (b + cx^2))^{-1+p} \right. \\ \left. \operatorname{AppellF1} \left[2, -p, -p, 3, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) / \left(\left(-b + \sqrt{b^2 - 4ac} \right) \right. \\ \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \left(-6a \operatorname{AppellF1} \left[2, -p, -p, 3, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] + \right. \\ \left. px^2 \left(\left(-b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[3, 1-p, -p, 4, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] - \right. \right. \\ \left. \left. \left(b + \sqrt{b^2 - 4ac} \right) \operatorname{AppellF1} \left[3, -p, 1-p, 4, -\frac{2cx^2}{b + \sqrt{b^2 - 4ac}}, \frac{2cx^2}{-b + \sqrt{b^2 - 4ac}} \right] \right) \right) \right)$$

Problem 1119: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^2 + c x^4)^p}{x} dx$$

Optimal (type 6, 152 leaves, 3 steps):

$$\frac{1}{p} 4^{-1+p} \left(\frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{cx^2} \right)^{-p} \left(\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{cx^2} \right)^{-p} (a + b x^2 + c x^4)^p$$

$$\text{AppellF1} \left[-2p, -p, -p, 1-2p, -\frac{b - \sqrt{b^2 - 4ac}}{2cx^2}, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2} \right]$$

Result (type 6, 497 leaves):

$$\begin{aligned} & \left(2^{-3-2p} c (-1+2p) \left(1 + \frac{b - \sqrt{b^2 - 4ac}}{2cx^2} \right)^{-p} x^2 \left(\frac{b - \sqrt{b^2 - 4ac}}{2c} + x^2 \right)^{-p} \right. \\ & \left. \left(\frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{c} \right)^{1+p} \left(\frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{cx^2} \right)^p \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) \right. \\ & \left. (a + b x^2 + c x^4)^{-1+p} \text{AppellF1} \left[-2p, -p, -p, 1-2p, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2}, \frac{-b + \sqrt{b^2 - 4ac}}{2cx^2} \right] \right) / \\ & \left(p \left(-\left(b + \sqrt{b^2 - 4ac} \right) p \text{AppellF1} \left[1-2p, 1-p, -p, 2-2p, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2}, \frac{-b + \sqrt{b^2 - 4ac}}{2cx^2} \right] + \right. \right. \\ & \left. \left(-b + \sqrt{b^2 - 4ac} \right) p \text{AppellF1} \left[1-2p, -p, 1-p, 2-2p, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2}, \frac{-b + \sqrt{b^2 - 4ac}}{2cx^2} \right] + \right. \\ & \left. \left. 2c (-1+2p) x^2 \text{AppellF1} \left[-2p, -p, -p, 1-2p, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2}, \frac{-b + \sqrt{b^2 - 4ac}}{2cx^2} \right] \right) \right) \end{aligned}$$

Problem 1120: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x^2 + c x^4)^p}{x^3} dx$$

Optimal (type 6, 166 leaves, 3 steps):

$$-\frac{1}{(1-2p)x^2} 2^{-1+2p} \left(\frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{cx^2} \right)^{-p} \left(\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{cx^2} \right)^{-p}$$

$$(a + b x^2 + c x^4)^p \text{AppellF1} \left[1-2p, -p, -p, 2(1-p), -\frac{b - \sqrt{b^2 - 4ac}}{2cx^2}, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2} \right]$$

Result (type 6, 516 leaves):

$$\begin{aligned}
 & \left(2^{-1-2p} (-1+p) \left(1 + \frac{b - \sqrt{b^2 - 4ac}}{2cx^2} \right)^{-p} \left(\frac{b - \sqrt{b^2 - 4ac}}{2c} + x^2 \right)^{-p} \left(-b + \sqrt{b^2 - 4ac} - 2cx^2 \right) \right. \\
 & \left. \left(\frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{c} \right)^p \left(\frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{cx^2} \right)^p \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) (a + bx^2 + cx^4)^{-1+p} \right. \\
 & \text{AppellF1} \left[1 - 2p, -p, -p, 2 - 2p, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2}, \frac{-b + \sqrt{b^2 - 4ac}}{2cx^2} \right] \Big/ \left((-1 + 2p) \right. \\
 & \left. \left(-4c(-1+p)x^2 \text{AppellF1} \left[1 - 2p, -p, -p, 2 - 2p, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2}, \frac{-b + \sqrt{b^2 - 4ac}}{2cx^2} \right] + \right. \right. \\
 & \left. \left(b + \sqrt{b^2 - 4ac} \right)^p \text{AppellF1} \left[2 - 2p, 1 - p, -p, 3 - 2p, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2}, \frac{-b + \sqrt{b^2 - 4ac}}{2cx^2} \right] + \right. \\
 & \left. \left. \left(b - \sqrt{b^2 - 4ac} \right)^p \text{AppellF1} \left[2 - 2p, -p, 1 - p, 3 - 2p, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2}, \frac{-b + \sqrt{b^2 - 4ac}}{2cx^2} \right] \right) \right)
 \end{aligned}$$

Problem 1121: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + bx^2 + cx^4)^p}{x^5} dx$$

Optimal (type 6, 164 leaves, 3 steps):

$$\begin{aligned}
 & -\frac{1}{(1-p)x^4} 4^{-1+p} \left(\frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{cx^2} \right)^{-p} \left(\frac{b + \sqrt{b^2 - 4ac} + 2cx^2}{cx^2} \right)^{-p} \\
 & (a + bx^2 + cx^4)^p \text{AppellF1} \left[2(1-p), -p, -p, 3 - 2p, -\frac{b - \sqrt{b^2 - 4ac}}{2cx^2}, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2} \right]
 \end{aligned}$$

Result (type 6, 504 leaves):

$$\begin{aligned}
 & \left(2^{-3-2p} c(-3+2p) \left(1 + \frac{b - \sqrt{b^2 - 4ac}}{2cx^2} \right)^{-p} \left(\frac{b - \sqrt{b^2 - 4ac}}{2c} + x^2 \right)^{-p} \left(\frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{c} \right)^{1+p} \right. \\
 & \left. \left(\frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{cx^2} \right)^p \left(b + \sqrt{b^2 - 4ac} + 2cx^2 \right) (a + bx^2 + cx^4)^{-1+p} \right. \\
 & \text{AppellF1} \left[2 - 2p, -p, -p, 3 - 2p, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2}, \frac{-b + \sqrt{b^2 - 4ac}}{2cx^2} \right] \Big/ \left((-1 + p)x^2 \right. \\
 & \left. \left(2c(-3+2p)x^2 \text{AppellF1} \left[2 - 2p, -p, -p, 3 - 2p, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2}, \frac{-b + \sqrt{b^2 - 4ac}}{2cx^2} \right] - \right. \right. \\
 & \left. \left. p \left(\left(b + \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[3 - 2p, 1 - p, -p, 4 - 2p, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2}, \frac{-b + \sqrt{b^2 - 4ac}}{2cx^2} \right] + \right. \right. \right. \\
 & \left. \left. \left(b - \sqrt{b^2 - 4ac} \right) \text{AppellF1} \left[3 - 2p, -p, 1 - p, \right. \right. \right. \\
 & \left. \left. \left. 4 - 2p, -\frac{b + \sqrt{b^2 - 4ac}}{2cx^2}, \frac{-b + \sqrt{b^2 - 4ac}}{2cx^2} \right] \right) \right) \right)
 \end{aligned}$$

Problem 1122: Result more than twice size of optimal antiderivative.

$$\int x^4 (a + b x^2 + c x^4)^p dx$$

Optimal (type 6, 138 leaves, 2 steps):

$$\frac{1}{5} x^5 \left(1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}} \right)^{-p} \left(1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}} \right)^{-p} (a + b x^2 + c x^4)^p$$

$$\text{AppellF1}\left[\frac{5}{2}, -p, -p, \frac{7}{2}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right]$$

Result (type 6, 457 leaves):

$$\begin{aligned} & \left(7 \times 2^{-2-p} c \left(b + \sqrt{b^2 - 4 a c} \right) x^5 \left(\frac{b - \sqrt{b^2 - 4 a c}}{2 c} + x^2 \right)^{-p} \right. \\ & \left. \left(\frac{b - \sqrt{b^2 - 4 a c} + 2 c x^2}{c} \right)^{1+p} \left(-2 a + \left(-b + \sqrt{b^2 - 4 a c} \right) x^2 \right)^2 (a + b x^2 + c x^4)^{-1+p} \right. \\ & \left. \text{AppellF1}\left[\frac{5}{2}, -p, -p, \frac{7}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) / \\ & \left(5 \left(-b + \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\ & \left(-7 a \text{AppellF1}\left[\frac{5}{2}, -p, -p, \frac{7}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] + \right. \\ & p x^2 \left(\left(-b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1}\left[\frac{7}{2}, 1-p, -p, \frac{9}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] - \right. \\ & \left. \left. \left. \left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1}\left[\frac{7}{2}, -p, 1-p, \frac{9}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] \right) \right) \right) \end{aligned}$$

Problem 1123: Result more than twice size of optimal antiderivative.

$$\int x^2 (a + b x^2 + c x^4)^p dx$$

Optimal (type 6, 138 leaves, 2 steps):

$$\frac{1}{3} x^3 \left(1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}} \right)^{-p} \left(1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}} \right)^{-p} (a + b x^2 + c x^4)^p$$

$$\text{AppellF1}\left[\frac{3}{2}, -p, -p, \frac{5}{2}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right]$$

Result (type 6, 457 leaves):

$$\begin{aligned} & \left(5 \times 2^{-2-p} c \left(b + \sqrt{b^2 - 4 a c} \right) x^3 \left(\frac{b - \sqrt{b^2 - 4 a c}}{2 c} + x^2 \right)^{-p} \right. \\ & \left. \left(\frac{b - \sqrt{b^2 - 4 a c} + 2 c x^2}{c} \right)^{1+p} \left(-2 a + \left(-b + \sqrt{b^2 - 4 a c} \right) x^2 \right)^2 \left(a + b x^2 + c x^4 \right)^{-1+p} \right. \\ & \left. \text{AppellF1} \left[\frac{3}{2}, -p, -p, \frac{5}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\ & \left(3 \left(-b + \sqrt{b^2 - 4 a c} \right) \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\ & \left. \left(-5 a \text{AppellF1} \left[\frac{3}{2}, -p, -p, \frac{5}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \right. \\ & \left. \left. p x^2 \left(\left(-b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{2}, 1 - p, -p, \frac{7}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] - \right. \right. \right. \\ & \left. \left. \left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{5}{2}, -p, 1 - p, \frac{7}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) \end{aligned}$$

Problem 1124: Result more than twice size of optimal antiderivative.

$$\int (a + b x^2 + c x^4)^p dx$$

Optimal (type 6, 133 leaves, 2 steps):

$$\begin{aligned} & x \left(1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}} \right)^{-p} \left(1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}} \right)^{-p} \left(a + b x^2 + c x^4 \right)^p \\ & \text{AppellF1} \left[\frac{1}{2}, -p, -p, \frac{3}{2}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}} \right] \end{aligned}$$

Result (type 6, 487 leaves):

$$\begin{aligned} & \left(3 \times 4^{-1-p} \left(b + \sqrt{b^2 - 4 a c} \right) x \left(\frac{b - \sqrt{b^2 - 4 a c}}{2 c} + x^2 \right)^{-p} \left(\frac{b + \sqrt{b^2 - 4 a c}}{2 c} + x^2 \right)^{-p} \right. \\ & \left. \left(\frac{b - \sqrt{b^2 - 4 a c} + 2 c x^2}{c} \right)^{1+p} \left(\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{c} \right)^{-1+p} \left(-2 a + \left(-b + \sqrt{b^2 - 4 a c} \right) x^2 \right)^2 \right. \\ & \left. \left(a + b x^2 + c x^4 \right)^{-1+p} \text{AppellF1} \left[\frac{1}{2}, -p, -p, \frac{3}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\ & \left(\left(-b + \sqrt{b^2 - 4 a c} \right) \left(-3 a \text{AppellF1} \left[\frac{1}{2}, -p, -p, \frac{3}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \right. \\ & \left. \left. p x^2 \left(\left(-b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{3}{2}, 1 - p, -p, \frac{5}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] - \right. \right. \right. \\ & \left. \left. \left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[\frac{3}{2}, -p, 1 - p, \frac{5}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \right) \end{aligned}$$

Problem 1125: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x^2+c x^4)^p}{x^2} dx$$

Optimal (type 6, 136 leaves, 2 steps):

$$-\frac{1}{x} \left(1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}\right)^{-p} \left(1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right)^{-p} \\ (a+b x^2+c x^4)^p \operatorname{AppellF1}\left[-\frac{1}{2}, -p, -p, \frac{1}{2}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right]$$

Result (type 6, 472 leaves):

$$-\left(\left(2^{-2-p} (b + \sqrt{b^2 - 4 a c})\right) \left(\frac{b - \sqrt{b^2 - 4 a c}}{2 c} + x^2\right)^{-p} (-b + \sqrt{b^2 - 4 a c} - 2 c x^2)\right. \\ \left.\left(\frac{b - \sqrt{b^2 - 4 a c} + 2 c x^2}{c}\right)^p (-2 a + (-b + \sqrt{b^2 - 4 a c}) x^2)^2 (a+b x^2+c x^4)^{-1+p}\right. \\ \left.\operatorname{AppellF1}\left[-\frac{1}{2}, -p, -p, \frac{1}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right]\right) / \left(\left(-b + \sqrt{b^2 - 4 a c}\right) x\right. \\ \left.(b + \sqrt{b^2 - 4 a c} + 2 c x^2\right) \left(a \operatorname{AppellF1}\left[-\frac{1}{2}, -p, -p, \frac{1}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] +\right. \\ \left.p x^2 \left(\left(b - \sqrt{b^2 - 4 a c}\right) \operatorname{AppellF1}\left[\frac{1}{2}, 1-p, -p, \frac{3}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right] +\right. \right. \\ \left.\left.\left(b + \sqrt{b^2 - 4 a c}\right) \operatorname{AppellF1}\left[\frac{1}{2}, -p, 1-p, \frac{3}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}}\right]\right)\right)\right)$$

Problem 1126: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x^2+c x^4)^p}{x^4} dx$$

Optimal (type 6, 138 leaves, 2 steps):

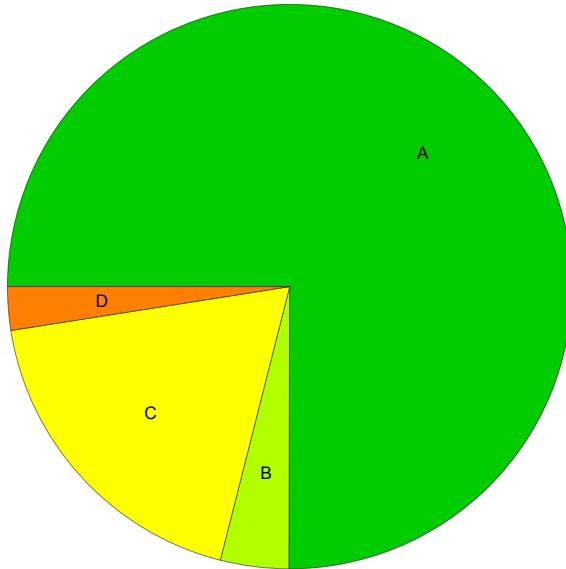
$$-\frac{1}{3 x^3} \left(1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}\right)^{-p} \left(1 + \frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right)^{-p} \\ (a+b x^2+c x^4)^p \operatorname{AppellF1}\left[-\frac{3}{2}, -p, -p, -\frac{1}{2}, -\frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}\right]$$

Result (type 6, 456 leaves):

$$\begin{aligned}
 & \left(2^{-2-p} c \left(b + \sqrt{b^2 - 4 a c} \right) \left(\frac{b - \sqrt{b^2 - 4 a c}}{2 c} + x^2 \right)^{-p} \right. \\
 & \quad \left. \left(\frac{b - \sqrt{b^2 - 4 a c} + 2 c x^2}{c} \right)^{1+p} \left(-2 a + \left(-b + \sqrt{b^2 - 4 a c} \right) x^2 \right)^2 \left(a + b x^2 + c x^4 \right)^{-1+p} \right. \\
 & \quad \left. \text{AppellF1} \left[-\frac{3}{2}, -p, -p, -\frac{1}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) / \\
 & \left(3 \left(-b + \sqrt{b^2 - 4 a c} \right) x^3 \left(b + \sqrt{b^2 - 4 a c} + 2 c x^2 \right) \right. \\
 & \quad \left(a \text{AppellF1} \left[-\frac{3}{2}, -p, -p, -\frac{1}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] + \right. \\
 & \quad \left. p x^2 \left(\left(-b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[-\frac{1}{2}, 1-p, -p, \frac{1}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] - \right. \right. \\
 & \quad \left. \left. \left(b + \sqrt{b^2 - 4 a c} \right) \text{AppellF1} \left[-\frac{1}{2}, -p, 1-p, \frac{1}{2}, -\frac{2 c x^2}{b + \sqrt{b^2 - 4 a c}}, \frac{2 c x^2}{-b + \sqrt{b^2 - 4 a c}} \right] \right) \right) \left. \right)
 \end{aligned}$$

Summary of Integration Test Results

1126 integration problems



- A - 845 optimal antiderivatives
- B - 44 more than twice size of optimal antiderivatives
- C - 209 unnecessarily complex antiderivatives
- D - 28 unable to integrate problems
- E - 0 integration timeouts